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MULTIPLE ELECTRON TRANSITIONS AND PRIMED SPECTRAL TERMS*

By Arthur Edward Ruark

ABSTRACT

Excitation of the lines of Mg^+ in low-voltage, low-current discharge. The stronger lines of Mg^+ appear at potentials equal to the ionization potential of Mg plus the excitation voltages predicted from their classifications. This proves that a single impacting electron can ionize an atom and raise a valence electron of the ion to a higher orbit.

Excitation of the pp′ lines of Mg and Cd in low-voltage, low-current discharge. Using a quartz spectrograph to analyze light from the force-free space between grid and plate, it was found that the Mg pp′ group (2776 to 2783 A) appears at current densities less than 0.2 milliampere per square centimeter. Since these lines are produced by electron transitions from an initial state in which there are two electrons in virtual orbits, this observation shows that the two electrons are moved to these orbits as a result of a single collision; lines which could be produced only by successive impact are entirely absent at much larger current densities. The pp′ group of Cd (2239 to 2329 A) appears at its quantum voltage, but is too faint to be detected at current densities which exclude the possibility of successive excitation.

Several lines of Cd, In, and Tl are classified by the use of primed terms.

Extension of the subordinate series of Cd: spectra of Cd below the ionization potential. The low-voltage arc in Cd yielded a number of new lines of the singlet and triplet subordinate series. In exposures taken below the ionization potential the lines of Cd are excited exactly at the voltage intervals predictable from the classification of the spectrum.

I. INTRODUCTION

From the theory of band spectra we have become accustomed to the idea that several simultaneous processes may contribute to the change of energy of an emitting molecule. Changes in both the rotational and oscillational states of the atom kernels may occur at the time of an electronic transition, and result in the emission of a single quantum corresponding to the total alteration in energy of the molecule.

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In atomic spectra we encounter the so-called "primed" terms corresponding to states in which two electrons are on excited orbits at the same time. Lines are known which must be produced by the simultaneous transition of both electrons to more closely bound orbits, though cases also occur such that one of the two retains its quantum numbers while the other jumps to its normal orbit.

This raises the question: Is it possible for a single impacting electron to move two electrons of an atom to higher orbits? Such a question naturally suggests others of a similar character. For example, we may ask, Can a single impacting electron ionize an atom and raise a valence electron of the ion to a higher orbit? In Sections II and III both questions are answered in the affirmative.

Before describing the experimental work we shall review briefly our knowledge of primed spectral terms and multiple electron transitions, in order that Section III may be better understood. The Paschen numeration of spectral terms is used, and the subscripts denote *inner quantum numbers.*

Thd idea of quantum transitions in which two electrons jump simultaneously was used by Grotrian[1] to explain a frequency difference of about 780 cm^{-1} which occurs a great number of times in the neon spectrum. This difference is approximately equal to the relativity separation of the L_{21} and L_{22} energy levels as calculated from Sommerfeld's formula. Suppose that an outer electron of neon has been raised to a virtual orbit, that is, one which is normally unoccupied. Transitions between *virtual* orbits give spectrum lines which Paschen[2] has classified. There are many lines which fall into series of the Ritzian type, but there are also lines whose wave numbers differ by ± 780 cm^{-1} from those of the Ritzian lines. These so-called non-Ritzian lines are presumably produced when a second electron passes between the L_{21} and L_{22} orbits at the same time that the first electron performs a jump of the usual type. It is very interesting that on this theory the transition of the L electron may occur *in either direction* while the other electron is passing toward its more closely bound orbit.

The detection of multiple electron transitions in neon depends on the fact that the energy contributed by the jump of the more closely bound electron is very small. The screening of the outside electron on a virtual orbit is practically the same, whether the more closely bound electron is on the L_{21} or the L_{22} level. Thus their separate contributions

[1] Grotrian, ZS. für Phys. *8*, p. 116; 1921.

[2] Paschen and Götze, Seriengesetze der Linienspektren, Berlin, 1922.

to the energy loss during a double transition are algebraically additive to a high degree of approximation. We may hope to find similar phenomena in the spectra of atoms with several valence electrons whose lowest energy levels are polyfold and comparatively close together. Lines should occur with wave numbers which are greater (or less) than those of prominent series lines by an amount approximately equal to the separations of the polyfold normal level. Up to the present no such line has been identified.

In 1922 the writer noticed three strong unclassified lines in the spectrum of neutral cadmium which have the separations of the triplet $2p$ energy levels of that element. All of these lines have wave numbers higher than the closest bound $2p$ term, namely $2p_0$. If we retain the convention that wave numbers increase as the energy of the atom decreases, and that the state of zero energy is that in which the valence electron is at infinity, all others being on their normal levels, then the initial energy level involved in the emission of these lines has a negative wave number. When the atom is ready to emit them it has a greater amount of energy than that corresponding to the first ionization potential. Similar negative terms have been discovered by Russell and Saunders[3] in the spectrum of calcium. Wentzel[4] has also called attention to such energy levels. The existence of negative terms is conclusive evidence that the two valence electrons are on excited orbits simultaneously. Russell and Saunders found that three negative terms and two positive terms of calcium form a series $2p - mp'$ with its limit at -13961 cm^{-1}. This shows that the same interpretation is to be extended to the positive terms involved.

There are two possible explanations of lines of the type $2p - mp'$:

(A) Two electrons perform quantum transitions simultaneously. In the initial state both valence electrons are on virtual orbits; one of them returns to its normal orbit, while the other performs a quantum transition to a $2p$ orbit.

(B) Only one electron performs quantum transitions. In the initial state both valence electrons are on virtual orbits; one of them has the total and azimuthal quantum numbers characteristic of the $2p$ energy levels, while the quantum numbers of the other need not be specified; the second electron returns to its normal orbit, $1S$, while the first *retains* its quantum numbers and at the end of the process is therefore on a $2p$ energy level.

[3] Phys. Rev., *22*, p. 201, 1923 and Astrophys. Jour. *61*, p. 38, 1925.
[4] Phys. Z. *25*, p. 182, 1924.

Wentzel[5] has suggested that the lines $2p-2p'$ of calcium, strontium and barium are produced in accordance with hypothesis B, while both electrons jump in accordance with hypothesis A, when $2p-3p'$, $2p-4p'$, etc., are emitted.

There has been much discussion of the probable position of the second valence electron in the p' and d' states of the alkaline earth atoms. Landé[6] believes it occupies an orbit of azimuthal number one; Green and Petersen[7] suggest an orbit of azimuthal number two, while Wentzel[8] pronounces in favor of a d orbit. The question need not detain us here, and will doubtless be settled in the near future by the discovery of other series of primed terms.

II. IONIZATION OF MG FOLLOWED BY EXCITATION OF THE ION

Millikan[9] has shown that, except in the case of helium, an α-particle detaches only one electron from an atom which it encounters. However, it is well known that a low-speed electron can produce a doubly charged ion at a single impact in all the metallic vapors which have so far been examined with regard to this point. In this section we shall show that a single electron can ionize a magnesium atom and raise the valence electron of the ion to a higher orbit.

The method used is to photograph the spectrum of a low-voltage magnesium arc run at such low current densities that all effects due to successive impact or to absorption of light followed by impact are imperceptible. Phenomena due to successive excitation vary roughly in intensity as the square of the current density, so that it proved unnecessary to use extremely small currents. The type of tube employed is shown in Fig. 1. Electrons accelerated by the potential difference between the tungsten filament and the nickel helix pass into the force-free space between helix and cylinder. The narrow tube at the end of the cylinder permits light from this region to reach the slit of the spectrograph. The tube is heated by an electric furnace to temperatures between 420° and 470° C. The performance of a tube of this kind is so dependent on its design that it may be well to give some details. The charge of magnesium must be so disposed that none of it can be seen through the quartz window; otherwise the window will become dirty

[5] Phys. Zeit. *24*, p. 104, 1923 and *25*, p. 182, 1924.
[6] ZS. für Phys. *27*, p. 149, 1924.
[7] Astrophys. Jour. *60*, p. 301, 1924.
[8] Phys. Zeit. *24*, p. 104, 1923 and *25*, p. 182, 1924. See also Laporte and Wentzel, ZS. für Phys. *31*, p. 335, 1925.
[9] Phys. Rev. *18*, p. 456, 1921.

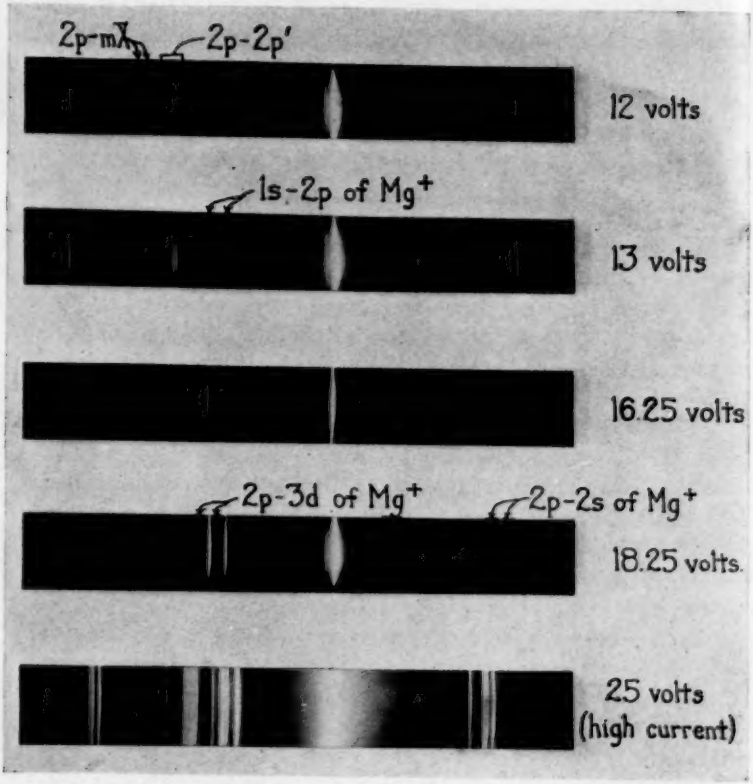


PLATE I. *Spectrum of ionized Mg, excited by single electronic impacts.*

very quickly. Heat from the filament prevents the metal from depositing in the middle of the chamber, so that the vapor pressure is far from uniform. Therefore, magnesium should be placed in both ends of the cylinder, so that two oppositely directed streams of vapor pass along it. Attention to this precaution greatly increases the light intensity attainable at a given temperature. It is best to make the cylinder so long that some of the magnesium is not in the region of the discharge; this maintains a *small* gas pressure in spite of the "clean-up" due to the discharge, and helps to keep the window free from metal.

A size E_2 Hilger quartz spectrograph was used, with exposures of two to twelve hours, depending on voltage. The only impurity lines were $1S-2p_2$ of Cd and Zn.

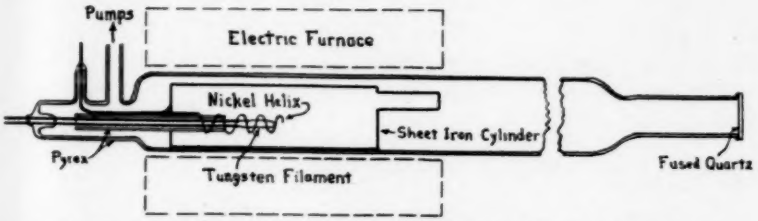


FIG. 1. *Three-electrode discharge tube for photographing the low voltage arc in magnesium.*

When the arc is run with a total current of 6 ma, corresponding to a current density of 0.2 ma/cm², the lines of Mg^+ appear at potentials equal to the first ionization potential of Mg (7.61 volts), plus the excitation voltages predicted from their classifications.

If conditions had been such that these lines were produced by successive excitation, they would have appeared slightly above the ionization potential. As a matter of fact, Foote, Meggers and Mohler[10] observed $1s-2p$, $2p-2s$, and $2p-3d$ of the ionized atom at 10 volts when using a current of 300 milliamperes.

Plate I shows some results of the present work. $1s-2p$ should appear at 12.01 volts. It is absent at 11, very faintly present at 12, and strongly developed at 13 volts. $2p-2s$ and $2p-3d$ have quantum voltages of 16.22 and 16.43 volts, respectively. They are definitely absent at 16.25 volts but are very strong at 18.25 volts. At 19.18 volts the very intense line 4481 A, $3d-4b$, should appear. It is absent at 19.25 volts though present at higher potentials. All other lines which could be excited at these voltages lie in inaccessible regions.

[10] Phil. Mag. *42*, p. 1002, 1921.

Thus the main features of Fowler's classification of Mg^+ are fully confirmed by these experiments, and we see that a single impact may produce ionization followed by excitation to an s, a p, or a d orbit.

III. EXCITATION OF THE PP' GROUPS OF MAGNESIUM AND CADMIUM

1. *Magnesium:* Discharge tubes of the type described above were used to determine the appearance voltages of the $2p-2p'$ group of magnesium under such conditions that successive excitation plays no rôle. This group, extending from 2776 to 2783A and having an excitation potential of 7.14 volts, is peculiar in certain respects. The $2p'$ levels have practically the same separations as the $2p$ levels, and it happens that the separations of the $2p$ levels are almost exactly in the ratio 2:1. The result is that three lines of the group nearly coincide with the lines $2p_{0,1,2}-5s$, while still another line, 2779.83A, is obliged to do double duty as $2p_2-2p_2'$ and $2p_1-2p_1'$. The group is shown below in accordance with the very accurate data of Green and Petersen:[7]

	(4)		
	2781.418		$2p_0'$
(6)	(10)	(4)	
2782.973	2779.834	2778.272	$2p_1'$
(10)	(6)		
2779.834	2776.691		$2p_2'$
$2p_2$	$2p_1$	$2p_0$	

Green and Petersen observed the line $2p_2-5s$ at 2781.292 A. The dispersion used in the present work does not suffice to separate this line from 2781.418 and the group appears to consist of five lines. Plate II shows some of the results. The given voltages are applied, the actual potentials being smaller. The group does not appear at 7.5 volts, but is quite strong in the 7.9 volt exposure, which, after applying initial velocity corrections, is actually a little below the ionization potential. The 9-volt exposure is definitely above the ionization potential.

These photographs show also a pair at 2768 A and 2765 A which first appears faintly in the 7.9 volt exposure. This pair is usually stated to be $2p_{1,2}-6D$. Green and Petersen have questioned this classification. There is much physical evidence to support their belief that this pair arises from a primed singlet term X, identical in position with $6D$.

This work proves that the lines $2p-2p'$ and $2p_{1,2}-X$ appear at their quantum voltages due to single electron impacts which move two electrons simultaneously to higher orbits. It will be understood

[7] Astrophys. Jour. *60*, 301, 1924.

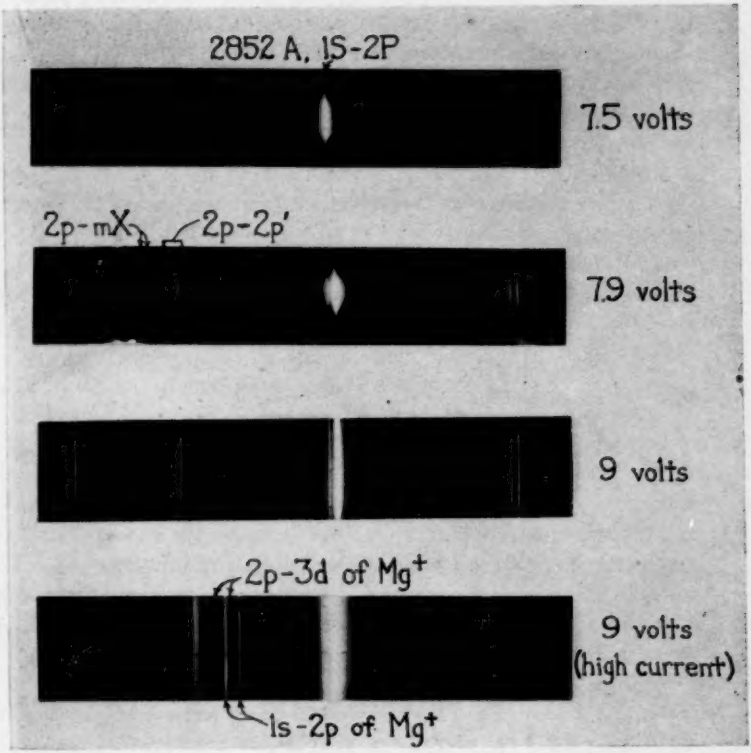


PLATE II. *Excitation of 2p−2p' and 2p−mX of Mg by single electronic impacts.*

that other methods of excitation may play a predominant part under other physical conditions. At high vapor pressures and current densities excitation by successive impact or by absorption followed by impact is probably the main factor in producing these lines. Lines of this type are in general easy to reverse. King[11] found that $2p-2p'$ of calcium reverses in furnace spectra at the same temperature as the diffuse triplet series. Strutt and Fowler[12] state that the Mg line at 2779 A, classified as $2p_2-2p_2'$ and $2p_1-2p_1'$, is produced when magnesium vapor is mixed with active nitrogen. Under certain conditions this is the strongest line of the group, so it may be that longer exposures would have brought out the remaining lines. This observation means that two electrons of magnesium can be raised to excited orbits in a collision of the second kind with an activated N_2 molecule.

The intensity distribution in the $2p-2p'$ group of Mg is worthy of attention. The low-current spectrograms of Plates I and II show a distribution which agrees fairly well with qualitative intensity rules. It will be seen that the strongest line is

2781 A, $2p_2-5s$ and $2p_1-2p_0'$

while the next in intensity is

2779 A, $2p_2-2p_2'$ and $2p_1-2p_1'$

The other three lines are very much weaker and are roughly equal in intensity.

By the kindness of Drs. Foote, Meggers, and Mohler, we include in Plate I and II some unpublished spectrograms which they obtained at much higher currents in 1921. In each plate the lowest spectrogram is taken from their work. Table 1 summarizes the intensity data for this group. The numbers given are visual estimates with the intensity of 2779 arbitrarily set equal to 10. King[13] found that these lines occur

TABLE 1. *Intensities of the pp' lines of Mg*

Voltage:	50	20	20	18	12	9	9	9.5	8.7	7.9
Current in m-amp: (I.A.)	high	high	25	5	5	high	3	2	1	4.7
2776.69	12	6	8	5	5	6	5	6	—	5
2778.27	9	4	12	4	4	4	5	5	5	5
2779.83	10	10	10	10	10	10	10	10	10	10
2781.41	9	9	11	8	8	9	15	12	20	20
2782.97	9	5	9	5	5	5	5	7	—	4

[11] Mt. Wilson Contributions No. 174, 1920.
[12] Proc. Roy. Soc., A, *86*, p. 105, 1911.
[13] Astrophys. Jour. *48*, p. 13, 1918.

in the electric furnace with approximately equal intensities. (2779 A is a little stronger than the others.)

2. *Cadmium.* It will be shown in Section IV that the $2p-2p'$ group of Cd lies between 2329 and 2239 A. This group was studied in a three-electrode tube of the same general type used for magnesium, at potentials ranging from 6 to 65 volts, and at current densities of the order of 0.5 milliampere per square centimeter. Filaments of oxide-coated platinum and of tungsten were used. It was seldom necessary to heat the tube above 180° C. The helix used as a heating coil was of fairly low resistance and carried a current of 20—25 amperes, so that its magnetic field served to concentrate the discharge along the line of sight.

Exposures ranged from 2 hours to 2½ days. The Seed 30 plates employed were covered with light machine oil to render them more sensitive to the far ultraviolet. Lyman[14] has recommended "Nujol" to sensitize plates for the region of the vacuum spectrograph, but Shoemaker and Busch's "Odorless Machine Oil" gives better results with Seed 30 plates down to 1850 A. No band spectra were observed at voltages below 45. The only impurities detected were Hg, Zn, and Tl.

In contrast with the case of magnesium, it was not found possible to excite the pp' group without the resonance lines of ionized cadmium also appearing faintly. It will be shown in Section IV that the excitation potential for this group is about ¼ volt above the first ionization potential, the $2p'$ terms being negative. For this reason the group behaves like very high members of a subordinate series. It is much fainter with respect to subordinate series lines than the corresponding group in Mg. Therefore the exposures required to bring it out become so long that the spark resonance lines also appear.

The conclusion is this: With a current density of about 2 milliamperes per square centimeter, the lines 2329 A, 2306 A, and 2239 A are present at 10 volts, that is, one volt above the first ionization potential of cadmium. We should expect to observe 2267 A, but it was obscured by the heavily overexposed resonance line 2288 A. A faint indication of 3298 A, $2P-2p_0'$, was seen on one plate, but we cannot state with certainty that it was present. It seems very unlikely that the appearance of these lines under such conditions is due to successive impacts, but no proof of this can be given.

It was thought worth while to study the changes in their intensity when the current density was varied over wide ranges. The method

[14] Science *58*, p. 48, 1923.

of procedure is to take a series of photographs at a given voltage, in which the product of current density by exposure time is roughly constant. As might be expected, the pp' group is considerably enhanced with respect to $1S-2P$ at high current densities. This shows that these lines can be produced by successive excitations.

IV. CLASSIFICATIONS OF LINES INVOLVING PRIMED SPECTRAL TERMS

1. *Cadmium*

The $2p$ levels of cadmium are as follows:

$$2p_2 = 40711.5$$

$$\Delta\nu_{21} = 1171.1$$

$$2p_1 = 41882.6$$

$$\Delta\nu_{10} = 541.9$$

$$2p_0 = 42424.5 \qquad 1713.0$$

There is a strong reversed triplet having these separations. All the lines of this triplet have higher wave numbers than the highest $2p$ level, so that the initial orbit, which we shall call $2p_1'$, has a negative wave-number:

$$2p_1' = -2206.6 \text{ cm}^{-1}$$

λ(I.A.)	Intensity	ν	$\Delta\nu$	Classification
2329.27	10*R*	42918.6		$2p_2-2p_1'$
			1170.0	
2267.46	5*R*	44088.6		$2p_1-2p_1'$
			543.1	
2239.86	5*R*	44631.7		$2p_0-2p_1'$
			1713.1	

$2p_1'$ has the inner quantum number $j=1$. We should expect to find other reversed lines, corresponding to initial orbits having other values of j. There is only one more unclassified reversed line which answers our purposes,

2306.61 A 5R 43340.3

Its Zeeman pattern is a triplet having 3/2 the normal separation,[15] and this fixes the character of the whole group. It is certainly of the type $2p-mp'$. The line 2306 A has a wave number 351.9 cm^{-1} smaller than that of $1S-2P$, while the unclassified line 3298 A, intensity 4. has a wave-number 352.6 smaller than that of $1S-2p_1$.

[15] Paschen, Ann. der Phys. *35*, p. 867, 1911.

The initial orbit lies below $2p_1'$ and probably has the inner number 0. We shall call it $2p_0'$.

$$2p_0' = -1457.35 \text{ cm}^{-1}$$
$$3298 \text{ A} = 2P - 2p_0'$$
$$2306 \text{ A} = 2p_1 - 2p_0'$$

A careful search has failed to reveal the remaining members of the $2p - mp'$ group.

2. *Indium and Thallium*

The elements on the right side of the third column in the periodic table have simple doublet spectra, but there are terms which do not belong to the usual sequences. We give here some classifications in the spectra of indium and thallium. Uhler and Tanch[16] noted that the indium lines 2957.02 and 2775.36 A have the separation $2p_1 - 2p_2$; $2p_1$ and $2p_2$ are the highest terms of the spectrum of neutral indium, so these lines must be classified as $2p_2 - a$ and $2p_1 - a$ respectively. There are two other pairs having the separation $2p_1 - 2p_2$, which require the introduction of a close pair of energy levels mp_1' and mp_2'. The collected results are given below.

Classification of Indium Lines

λ(I.A.)	Intensity	ν	Classification	Terms
2957.02	2*R*	33808.0	$2p_2 - a$	$a = 10647.2$
2775.36	1	36020.7	$2p_1 - a$	
3051.19	1	32764.7	$2p_2 - mp_1'$	$mp_2' = 11689.55$
3051.07	4	32765.9	$2p_2 - mp_2'$	$mp_1' = 11690.80$
2858.19	1	34976.9	$2p_1 - mp_1'$	
2858.08	1	34978.2	$2p_1 - mp_2'$	

Only six lines of neutral indium are left unclassified.* Three of them are reversed, and therefore involve the energy levels $2p_2$ or $2p_1$, so there are at least three more terms which do not belong to the four main sequences. Grotrian[17] obtained the line 2836.90 A in absorption at a temperature of 1100° C, while lines involving the orbit $2p_2$ first make their appearance at about 800°. This observation makes it probable that 2836 A is of the type $2p_2 - \beta$, where $\beta = 9215.9$ cm^{-1}. Grotrian surmised, however, that this line is due to a transition of one of the closer bound electrons of the valence shell of indium. There is reasonably good evidence against this supposition. The fine structures of

[16] Uhler and Tanch. Astrophys. J. *55*, 291, 1922.

* The lines 3610.50 and 3261.06 listed by Exner and Haschek are due to cadmium.

[17] Grotrian, Zs. für Phys. *12*, p. 218, 1922.

2957 ($2p_2-a$), and of 2836 A, were photographed by Uhler and Tanch. The frequency-difference 0.509 cm^{-1} occurs in both fine structures, so that the lines should have one term in common, namely $2p_2$.

In thallium, we have the pair,

λ(I.A.)	Intensity	ν	Classification	Term
2671.10	2*	37426.7	$2p_2-x$	$x=4044.4$
2210.73	2R	45219.8	$2p_1-x$	

* Shaded toward short wave lengths.

The pair 2669.95 and 2209.9 A also has the separation $2p_1-2p_2$, and would require an orbit at 4028.7, but it is possible that 2209.9 does not belong to the arc spectrum. Examination of a spectrogram obtained by Mohler and Ruark[18] shows that at least one of the lines 2671 and 2669 A occurs in the low voltage arc below the ionization potential of thallium.

In the case of these third group elements, we do not know what physical characteristics of the atom correspond to the existence of the new energy levels. In the present state of our knowledge it is possible that they may not be primed terms but are really parts of a system of quadruplets. However, this seems improbable.

V. EXTENSION OF THE SUBORDINATE SERIES OF CADMIUM; SPECTRA OF CADMIUM BELOW THE IONIZATION POTENTIAL

The low voltage arc is an efficient source for the production of subordinate series lines. The sharp and diffuse singlets have been extended as shown in Table 2, while Table 3 gives new members of the sharp and diffuse triplet series. Lines marked with an asterisk have been observed before but are classified here for the first time; the wave lengths of other

TABLE 2. *Sharp and diffuse cadmium singlets*

λ(I.A.)	Intensity	ν	Classification	Running Term
3661.98*	—	27299.9	$2P-9S$	1546.7
3621.3	1	27606.8	$10S$	1239.8
3591.5	0	27835.8	$11S$	1010.8
3570.4	0	28000.2	$12S$	846.4
3774.8*	—	26483.7	$2P-7D$	2362.9
3695.9	6	27049.7	$8D$	1796.9
3645.0*	4	27427.3	$9D$	1419.3
3608?	—	—	$10D$	—
3582.7*	1	27904.1	$11D$	942.5
3563.2	1	28056.0	$12D$	782.7
3548.0	0	28177.0	$13D$	669.6

* Lines previously observed but not classified.

[18] Mohler and Ruark, J. Opt. Soc. Am. and R. S. I. 7, p. 819, 1923.

lines are probably correct to 0.2 A, judging by the agreement with calculated values.

The treatises of Fowler and of Paschen and Götze on series spectra are not in agreement as to the lines $2p_{0,1,2}, -7s$ and $2p_2-8s$. A careful study of the intensities of these lines supports Fowler's arrangement. The line listed by Paschen as $2p_2-7s$ may be due to Zn; his $2p_1-7s$ is really $2p_2-11s$, while 2474.15 A is much too weak to be $2p_0-7s$. The line which he calls $2p_2-8s$ is really $2p_1-6s$.

TABLE 3. *Sharp and diffuse cadmium triplets*

λ(I.A.)	Intensity	Classification	Running term	Remarks
2447.47	1	$2p_0-8s$	1576.8	
2533.91	3	$2p_2-9s$		Listed in Fowler's report
2480.72	2	$2p_1-9s$	1257.0	" " " "
2428.31	1	$2p_0-9s$		
2519.04+	–	$2p_2-10s$		Fused wtih $2p_0-6s$
2446.68	1	$2p_1-10s$	1023.2	
2414.65	0	$2p_0-10s$		Fused with $2p_1-14d$
2507.84*	–	$2p_2-11s$		
2436.25*	–	$2p_1-11s$	849.2	
2404.56	0	$2p_0-11s$		
2499.79	0	$2p_2-12s$		
2408.36*	n1	$2p_0-11d$	920.3	
2400.06*	1	$2p_0-12d$	771.6	
2393.42*	0	$2p_0-13d$	658.1	
2419.64+	–	$2p_1-14_0$		Covered by $2p_0-10d$
2388.16	0	$2p_0-14d$	566.7	
2485.61	1	$2p_2-15d$		
2415.24+	0	$2p_1-15d$	492.1	Fused with $2p_0-10s$
—	—	$2p_0-15d$		
2481.81	0	$2p_2-16d$		
2411.70	0	$2p_1-16d$	430.6	
—	–	$2p_0-16d$		
2478.89	–	$2p_2-17d$	383.1	

* Lines previously observed but not classified.
\+ Wave length calculated.

Some of the plates obtained in this study at high current density and vapor pressure show reversals of the sharp triplet series up to $2p_2-7s$, $2p_1-8s$, and $2p_0-7s$ while the diffuse triplet lines were reversed as far as $2p_2-11d$, $2p_1-11d$, and $2p_0-10d$. This provides confirmation of the classification of these lines as given by Fowler.

These series afford an illustration of the fact that in general more members are observed in a diffuse series than in the corresponding sharp series. This may be understood in a rough way by considering

the dimensions of the s and d orbits of high total quantum number, without any reference to the correspondence principle.

The lines $2P-5D$ and $2P-6D$ at 4140.5 A and 3905.1 A are abnormally faint. Their intensities are 1 and 5, respectively. Similar anomalies are found in the singlet systems of Ca and Ba. $2P-10D$ is hidden by the broad cadmium line at 3610.50.

A series of exposures was obtained below the ionization potential of Cd. Subordinate series lines and intercombination lines appeared exactly at the voltage intervals predictable from the classification of the spectrum. These results are discussed in detail in another publication.[19]

VI. DISCUSSION OF RESULTS

In conclusion, we may state that the spectrum of neon, molecular band spectra, and the phenomena of primed spectral terms yield excellent evidence that multiple electron transitions actually exist. We have seen that the final multiplicity of the orbits of a valence electron is not in general altered when another valence electron passes to excited orbits, and the inner quantum numbers are the same as for levels which belong to the usual sequences. This opens the question, how do the spectral levels of alkaline earth atoms coalesce to form the doublet levels of the once-ionized atoms, as the second valence electron is removed to successively higher orbits? This progressive alteration may be manifested by the changing *separations* of primed spectral levels, or by the entire absence of certain lines of the primed groups. Another mode of attack is furnished by the Zeeman effect. When the second valence electron is on orbits of small binding energy we may expect that the rules of formal quantum theory are not precisely obeyed. For example, the magnetic separations of the spectral levels might be of a hybrid nature, intermediate between those characteristic of doublet levels and those appropriate to triplets and singlets. Experiments to test this conclusion are much to be desired.

I wish to express my hearty thanks to Professor J. S. Ames and Professor R. W. Wood, and to Drs. F. L. Mohler and P. D. Foote for their unfailing interest and helpful suggestions; and to Mr. R. L. Chenault for a great deal of aid in obtaining spectrograms.

BUREAU OF STANDARDS
AND
THE JOHNS HOPKINS UNIVERSITY.

[19] Ruark and Chenault, J.O.S.A. & R.S.I., *10*, p. 653, 1925.

Theory of the Striated Discharge.—It may be accepted as a fact of observation that, in the luminous portion of each striation of a striated discharge, there is an excess of negative space-charge; and it is natural to assume that this is due primarily to the stoppage of electrons by inelastic impacts against atoms. Günther-Schulze's chief addition to this simple interpretation consists in remarking that the space-charge originating thus will retard the oncoming electrons in such a way that their drift-speed will be reduced in comparison with their zigzag uncoordinated motion; in other words, they will bounce sidewise and to-and-fro in this luminous region without getting onward and out of it as rapidly as they otherwise would, and this will accentuate the space-charge effect, and the chance of further inelastic impacts, and presumably contribute to the sharpness and permanence of the striations. He also emphasizes that although the loss of energy suffered by an electron at an elastic impact with an atom is small, yet in a gas at moderate pressure the impacts are numerous enough to prevent the electrons from attaining more than an unexpectedly modest terminal speed, and attaining it unexpectedly slowly. Thus in hydrogen at 1 mm pressure, assuming the mean free path of the electron to be 0.8 mm and all the impacts to be elastic, a field of 1 volt/cm would not impart more than 1.6 equivalent volts of energy to an electron, no matter how far it went; and it would have to go 2.5 cm to acquire 90 per cent of that much energy. This and corresponding values for other gases are calculated from Hertz's equations. These facts are adequate to explain why the P. D. between successive striations is generally well above the resonance potential of the gas. The author brings out the paradoxical fact that, as the electron gains speed from the field, its forward drift-speed falls off by virtue of the increase in its velocity of uncoordinated motion. To explain why electronegative contaminations are required to produce striations in most gases, Günther-Schulze supposes that the resonance potentials are so numerous and so indistinct that the necessary sharp demarcations between regions of elastic and regions of inelastic impacts cannot be produced unless electronegative molecules start the accumulation of negative space-charge by catching electrons and holding them. In general it appears that while plausible theories can easily be constructed from considerations of this sort, they cannot be made forcefully convincing without a more profound statistical analysis than has yet been made.—[A. Günther-Schulze, Reichsanstalt; ZS. f. Phys. *31*, pp. 1-13, 1925].

KARL K. DARROW

THE STRUCTURE OF THE RED LITHIUM LINE λ 6708

By J. B. Green

ABSTRACT

The Structure of the Red Lithium Line λ6708.—The lithium line λ6708 was studied under varying conditions of pressure and current in a vacuum arc, and the Lummer pattern was found to be a simple doublet for low vapor densities. As the vapor density increased, each of the components split into two parts, which are attributed to reversal. The two middle components then merged, the distance between dark centers remaining constant, and then, as each of the reversed lines broadened, the merged components became fainter and finally disappeared when the vapor density became quite high, the pattern then resembling that of a single broadly reversed line. McLennan and Ainslee's wide quadruplet pattern must be attributed to incipient Stark effects.

The red lithium line has been studied by Woodward,[1] King,[2] Zeeman[3] (in absorption), and Kent,[4] and is generally accepted as being a doublet with separation .151Å. More recently, McLennan and Ainslie[5] studied the line using a vacuum arc as a source of radiation. They found a quadruplet pattern with the separations .128, .173, and .165A. This quadruplet pattern was attributed by the authors to the two isotopes of lithium, each radiating its own doublet. However, the separation of the two pairs of lines has no theoretical foundations, and the interpretation has been criticized because the intensities of the pairs attributed to the two isotopes do not correspond to the percentages of the isotopes as found by Aston.

The work was carried on by Ainslie[6] with a slightly different source, and he found that the structure of the line was quite variable, being either a doublet, a triplet, or a quadruplet, depending on conditions. However, the conditions under which he worked were not easily controllable. Indeed, he was at a loss to interpret his results in terms of these conditions, and it was with this aim in view that the present investigation was carried out.

APPARATUS

The apparatus used was the vacuum arc used by Petersen and Green[7] in their investigation of the pressure shift of several magnesium lines.

[1] Woodward, Ap. J., *41*, p. 169; 1915.
[2] King, Ap. J., *44*, p. 300; 1916.
[3] Zeeman, K. Akad. Wet. Amst., Proc. *15*, pp. 1130-1, 1913.
[4] Kent, Ap. J., *41*, p. 337; 1914.
Phys. ZS., *15*, p. 383; 1914.
[5] McLennan and Ainslie, Roy. Soc. Proc., A. *101* p. 342; 1922.
[6] Ainslie, Trans. Roy. Soc. Canada, III, *18*, p. 137; 1924.
[7] Peterson and Green, Ap. J., 1925.

In the present investigation with lithium, a hole was bored in the magnesium rod, and packed tightly with lithium chloride, which had been previously heated to drive off the water of crystallization. The pressure was controlled by allowing hydrogen to enter the tank and pumping continuously. The current could be varied from 4.5 to 15 amperes by means of a bank of water-cooled resistances.

The light from the negative electrode was focused by means of a Goerz Dagor *f* 6.3 lens on the slit of a Hilger constant deviation spectroscope with photographic attachment. A Lummer plate was placed between the collimator and prism. The constants of the plate yield for the wave length difference between successive orders $\Delta\lambda = .411$ A.

OBSERVATIONS

The observations were carried out in the following manner. The end of the rod (circular cross-section) was projected on the slit, with the rod considerably longer (5−8 mm) than the Marquardt tube, and photographs were taken for currents of about 5, 7.5, 10 and 12.5. amperes before the arc began to burn inside of the tube. Exposures were 3, 2, 1½, and 1 minute, respectively.

Using a pressure less than .3 mm, it was found that regardless of the current, the pattern of the λ6708 line was a simple doublet whose separation was constant within the limits of error of measurement, the short wave length component being about twice as bright as the long wave length.

The current was then maintained at 7.5 amperes. As the arc began burning in the pit, each of the lines broke up into two, the separation of the outer components being about .26 A, and the distance between dark centers about the same as the separation of the simple doublet. As the arc penetrated further into the tube the two inner components merged and the separation of the outer components increased to about .30 A, the distance between the dark centers being not appreciably different from the separation of the simple doublet.

Then, with the deep pit the current was increased to 12.5 amperes and photographs were taken at intervals of 2.5 amperes, down to about 5 amperes, allowing two minutes' pumping between exposures, and the pattern changed from the triplet with a fuzzy middle component to a quadruplet with the inner lines just barely resolved into two. At 5 amperes the two sets of components were quite widely separated, the pattern having the appearance of two reversed lines. The distance

between dark centers again was not noticeably different from the doublet separations.

The arc was then opened, and just a slight amount of lithium chloride placed in the end of the magnesium rod, and using a deep pit, photographs were taken every fifteen minutes, pumping all the while. The first photograph showed a quadruplet with the middle pair just barely resolved. The next pattern gave the appearance of two reversed lines. The third and fourth were simple doublets. The separation of the dark centers of the first two photographs was equal to the doublet separation.

Running the arc at higher pressures consumed the magnesium rod much faster and no simple doublet pattern could be produced above 1.5 mm pressure. Under most conditions using higher pressures, the pattern was a triplet with fuzzy middle component which gradually disappeared as the vapor density increased, and finally appeared as a single reversed line with two broad wings having a separation of about .192 A.

When the pressure exceeded 5 mm and the current 10 amperes, the orders appeared to overlap and the pattern seemed to be like that observed by McLennan and Ainslie.[5] It was noticeable, then, that λ6103, which presented a quite sharp doublet pattern under other conditions, now became extremely diffuse and hardly resolvable as a doublet.

SUMMARY

The patterns presented by the lithium line λ6708, as seen by Ainslie[6] and the author, can mostly be explained by self-reversal due to increased density of emitting vapor. With small vapor densities, the line appears as a simple doublet. As the vapor density increases, each of the members of the doublet appears to reverse, the reversals becoming wider and wider, until the two inside components of the quadruplet pattern thus produced merge, and as the vapor density increases, this middle component of the triplet pattern thus seen gradually disappears, because of the overlapping of the two reversals, leaving a pattern with the appearance of a single reversed line with broad wings.

Under very high electrical stimulation, the pattern appears to be a very wide quadruplet, probably because the two reversed lines are separated by incipient Stark effects due to interatomic fields.[8] The fact that Zeeman finds a doublet in absorption is our best evidence that a doublet is the normal pattern emitted by the atom.

[8] Stark, Jahr. d. Rad. u. El., *12*, p. 349; 1915.

In conclusion, the author wishes to acknowledge his indebtedness to Dr. Max Petersen of New York University, who suggested the problem and was himself prevented from carrying it out to solution, and to the Department of Physics for the facilities extended during its progress.[9]

UNIVERSITY OF WISCONSIN,
MADISON, WISCONSIN,
FEBRUARY 15, 1925.

Magnetic Moment of the K electrons of heavy Atoms.—It is pointed out that, owing to the dependence of mass on speed for electrons moving so rapidly and so closely to a highly-charged nucleus as the *K* electrons of heavy atoms, the ratio of magnetic moment to angular momentum should have, not the classical value $e/2mc$, but a value which in extreme cases may be 20 per cent less. Such theories of the anomalous Zeeman effect and the structure of multiplets as involve the assumption that this ratio is, not indeed $e/2mc$, but some small integer multiple of this value, require an explanation not merely of the multiplying factor, but also of the reason why the factor should be applied to the value $e/2mc$ without taking account of the relativity correction. This injures the acceptability of the theories. [W. Pauli, Hamburg; ZS. f. Phys. *31*, pp. 373-385; 1925]. KARL K. DARROW

Recrystallization of cold-rolled Silver produced by Annealing.—When a cold-rolled sheet of silver is annealed, x-ray analysis at various stages shows that the eventual random distribution of the crystals is proceeded by a stage in which there is neither random orientation, nor, on the other hand, the same preferential orientations as in the cold-rolled metal, but a tendency to preferential orientations distinct from those in the cold-rolled metal. Evidently the new crystals which form and grow during the annealing are not quite indifferent to direction, but tend to group themselves in certain orientations which are not necessarily those preferred by the original crystals, although related to them. If the annealing is carried out at a moderately low temperature such as 150°C, this "intermediate" stage persists indefinitely without giving way to a stage of random distribution in orientation.—[R. Glocker, Stuttgart; ZS. f. Phys. *31*, pp. 386-410, 1925].

KARL K. DARROW

[9] Note added March 4: Since reporting on this work at the meeting of the Physical Society at New York, a paper has appeared by Royds in the Proc. Roy. Soc., *107*A, p. 360, 1925, observing a similar phenomenon in the case of the green line of thallium λ5350, although here the more refrangible component reverses before the other, and hence no quadruplet pattern is observed. He also mentions λ6708 of lithium, though not going into detail, and mentions the fact that λ6103 of lithium also presents a triplet pattern under certain conditions. This last statement has not been verified by the author.

PHOTOGRAPHIC INVESTIGATION OF SCATTERED X-RADIATION

By P. A. Ross

In recent papers Barkla and Dauvillier have published results not in agreement with some of the writer's experiments on the scattering of x-radiation. The elusive nature of the Compton effect together with its great theoretical importance makes it desirable to publish here the spectrograms shown at the Washington meeting of the American Physical Society in December 1924 and to state the degree of accuracy of the results and outline some of the precautions needed in such work.

The ionization spectrometer has recently been used by Compton, Duane and others for the study of scattered x-rays. Such an investigation demands extreme sensitivity for the detection of the faint scattered radiation and the most careful shielding from stray radiation and disturbing electric fields.

The photographic method which possesses certain great advantages as well as disadvantages has been used by Ross, Becker and Ross and Webster. The great advantage of this method is the cumulative effect of exposure on the photographic plate allowing the use of higher resolving power and work with radiation scattered by elements of high atomic number where the absorption makes the available scattering volume of the metal extremely small. The disadvantage of long exposure (100 to 400 hours at 1 to 1.5 kilowatts input with a standard water cooled molybdenum Coolidge tube) is largely offset by the fact that the photographic outfit may be so well protected by simple automatic controls and safety switches that it will require but a few minutes attention each day. The cost of current and deterioration of the expensive tube during the long run is largely offset by the low cost of the simple equipment and the fact that the operator's time is free for other work. The time and cost per picture may be greatly reduced by placing a number of spectrographs about one tube as in the Hull powdered crystal apparatus. Even the lack of direct quantitative measure of relative intensity of the lines may be overcome by a photometric study of the plates.

Fig. 1 is a diagram of the watercooled molybdenum tube within its cylindrical lead box as used in taking spectrograms 1 to 7 inclusive. The axis of the tube was vertical and exposures of from 100 to 200

hours were given to a battery of spectrographs surrounding the tube. The spectrographs were of the single slit fixed crystal type. The large dispersion spectrogram No. 9 was taken with a Seeman type spectrograph and No. 10 with a modified Seeman with the slit at one edge of the crystal.

The current source was an ordinary large size x-ray induction coil driven on 220 volts, 60 cycle current and giving about 50 kilovolts peak. The middle point of the transformer secondary was grounded and the x-ray tube target cooled by water from a 30 gallon tank insulated on a glass legged table, the circulation being maintained by an insulated

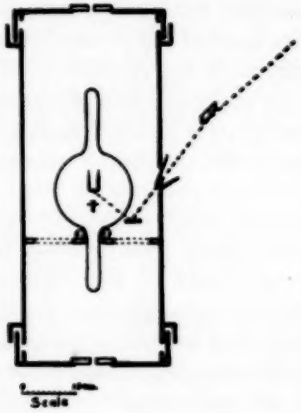


FIG. 1

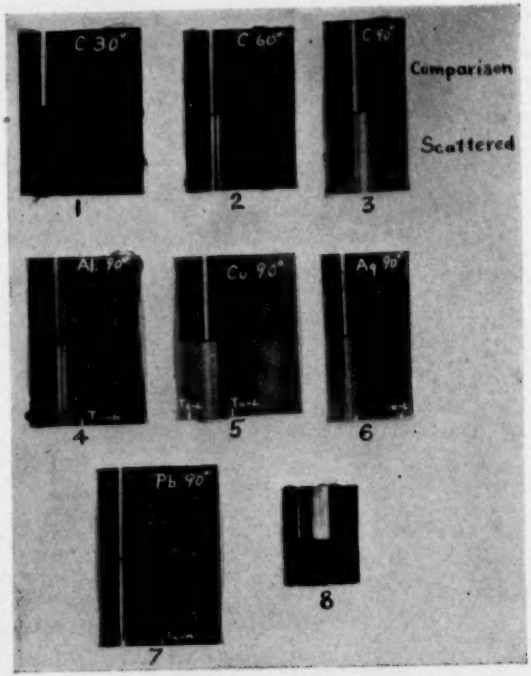
centrifugal pump. A water trigger device opened the primary switch if the circulation of the water stopped. The tube was run at about 30 milliamperes and an overload switch opened the primary circuit if the tube current reached 35 m.a.

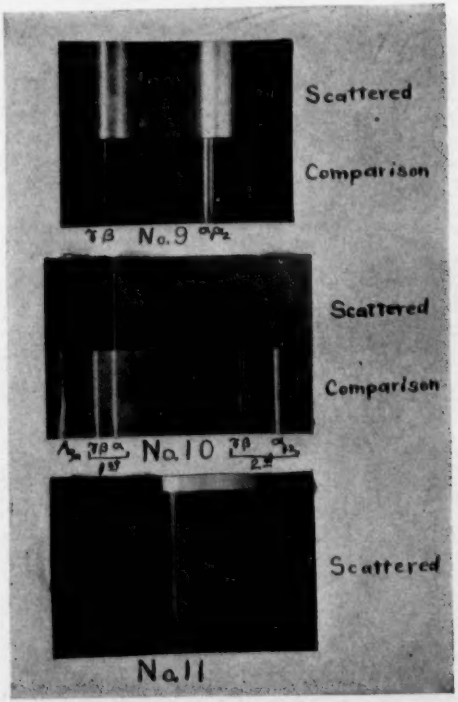
The results may be summarized in a table as given on page 219.

Spectrograms 10 and 11 were taken by the writer and Dr. D. L. Webster during the autumn of 1924. All the others were taken by the writer during the spring of 1923.

Numbers 1 to 7 inclusive show the decrease in relative intensity of the modified to the unmodified line as the atomic number increases, and, in the case of graphite at least, the decrease of relative intensity of the modified line at small scattering angles.

Spectrum No. 8 is an enlargement of a picture taken of the green mercury line λ5461A, as resolved by a Lummer-Gehrcke interferometer.


C 30°
C 60°
C 90°
Comparison
Scattered
1
2
3
Al. 90°
Cu 90°
Ag 90°
4
5
6
Pb 90°
8
7


Scattered
Comparison
No. 9
Scattered
Comparison
No. 10
Scattered
No. 11

The upper half is the direct radiation and the lower half the radiation as scattered at 180° by a paraffin block. The main radiation is here evidently unmodified and any modified component, if it exists, must be relatively very faint. This is in line with some rather incomplete ionization data taken by the writer with Dr. D. L. Webster showing that tungsten K radiation gives an extremely strong modified line and with some preliminary photographs with copper K radiation showing that the modified line, if it exists, is very weak.

TABLE 1.

No.	Source	Scattering Substance	Angle of Scattering	Measured Shift	Computed Shift
1	Mo. α	Graphite	$30° \pm 5°$	$.004 \pm .001$ Å	.00324Å
2	"	"	$60° \pm 5°$	$.013 \pm .001$	.0121 A
3	"	"	$90° \pm 5°$	$.025 \pm .001$	.0242 A
4	"	Aluminum	$90° \pm 5°$	$.0240 \pm .001$	"
5	"	Copper	$90° \pm 5°$	$.0240 \pm .001$	"
6	"	Silver	$90° \pm 5°$	$.0240 \pm .001$	"
7	"	Lead	$90° \pm 5°$	$.0240 \pm .001$	"
8	Green Hg. λ5461 A	Paraffin	$180° \pm 5°$	$0. \pm .009$	.0484 A
9	Mo. α	Graphite	$90° \pm 5°$	$.0235 \pm .0003$	.0242 A
	" α_2			$.0235 \pm .0003$	"
	" β			$.0239 \pm .0002$	"
10	Mo. β (1st order)	Aluminum	$90° \pm 5°$	$.0240 \pm .0005$	"
	Mo. β "	"		$.0240 \pm .0007$	"
	Mo. α (2nd order)	"	$120° \pm 5°$	$.033 \pm .002$	"
	Mo. β "			$.035 \pm .001$	"
11	Mo. α	Sulphur	$90° \pm 5°$	$.0242 \pm .0005$	"

The increased breadth of the scattered lines is more apparent than real. In the case of the modified line it is chiefly due to lack of a definite homogeneous angle of scattering. The increased breadth of the unmodified line is probably partly due to the overlapping of cones of rays from various parts of the scattering block. With a point source, such as a focus tube, the spectrum line is given by the intersection of the photographic plate with a very flat cone of rays with its apex at the image of the focal spot back of the plane of the crystal face. Such a line is necessarily slightly curved. With a scattering block the family of cones from the large scattering area produce a narrow band with hazy edges rather than a line. The increased breadth may also be partly due to lack of exact parallelism between the slit and the crystal face.

This is more important in work with scattered and fluorescent radiation than with direct rays from a small source. With a point source, lack of parallelism would merely result in an inclined and slightly broadened line but with a large radiator crossfire would superpose inclined lines originating from various heights on the source into a much broadened but erect line.

Another source of possible trouble in work with an area rather than a point source is the effect of a misplaced element of the crystal face. With a point source such a flaw would produce an easily recognized spot or short line on the spectrogram and the flaw could be located and eliminated. With a source of considerable area crossfire from various heights on the radiator would draw the point out into a false line hard to distinguish from a true line.

These sources of possible error are not peculiar to the photographic method but are as important in ionization work, and the only safe method is to use the nearest perfect crystals obtainable and to repeat any suspicious results with other crystals or different portions of the same crystal. In fact the photographic tests for such defects are highly advisable as preliminaries to any ionization work. These precautions were, of course, taken in the work presented here.

STANFORD UNIVERSITY,
CALIFORNIA

Influence of occluded Gas on the electrical Behavior of metals.—Janitzky reports that he succeeded in making it quite impossible for an electron-current to flow across a highly-evacuated tube from a hot filament to a platinum disc anode (the filament in the center of the tube, the disc distant about 4 cm with its leading in wire sheathed in a glass capillary) merely by driving out the gas from the disc by long-continued intense electron bombardment. In another tube where there were two such discs, he thus made it impossible for current to proceed to one, although it continued to flow to the other symmetrically-placed one. He thinks that the metal becomes a perfect insulator when all of the gas is driven out of it, but cannot explain why this does not happen in all Coolidge tubes and audions, unless because the mass of the metal in such tubes is too great for the de-gassing ever to be perfect. He also reports that if two electrodes of the same material, one de-gassed and the other untreated, are pressed together, a current of positive charge flows from the former to the latter, remaining of the order of 10^{-7} amp. for twenty minutes or more. This he considers a measurement of contact P. D., which seems doubtful.—[A. Janitzky; ZS. f. Phys. *31*, pp. 277-295; 1925].

KARL K. DARROW

INSTRUMENT SECTION

TWO NEW SELENIUM CRYSTAL-BRIDGES

By A. M. McMahon and F. C. Brown

ABSTRACT

The construction of two selenium bridges, for studying the electrical properties of the well known acicular crystals of this element, is described. The bridges are distinguished by the devices employed to apply the pressure necessary for good contact between the crystals and a form of metallic grid used in both. They have been designed to eliminate difficulties of insufficient sensitiveness and an unsteadiness experienced by previous workers with certain phases of the light effect.

The characteristics of the bridges are such that, together, they have a wide range of usefulness. A screw type of pressure device provides a general utility bridge, while a gravity type results in an arrangement which is more suitable for the purpose of exact measurement. When the temperature is maintained constant, variations in the dark current through the first bridge are of the order of 10^{-7} ampere where the potential drop is two volts. In the second bridge they may be made a hundred times smaller. The grid feature, common to both bridges, brings about a change of current as much as a thousand times that obtained by illuminating a single crystal.

A theory of the unsteadiness, observed in the electrical resistance of selenium crystals by several investigators, is presented. It is believed that a steady condition can be secured only when the pressure used to make contact with the crystals is such that their elastic limit is not reached. A number of facts, collected during the development of the bridges mentioned above, are advanced in support of this view. A practical method for complying with the condition is given.

The study of the light-sensitive properties of individual crystals of metallic selenium was initiated by one of the writers in 1914 with the preparation of single crystals of suitable size.[1] Since then many investigations have been undertaken following various methods of mounting already described. Wave-length sensibility curves have been obtained for separate crystals.[2] The influence of mechanical pressure and potential difference upon their sensitiveness has been studied.[3] A transmitted effect has been discovered.[4] However, owing to the rather high electrical resistance of the crystals, many interesting experiments, particularly those involving illumination by monochromatic light, either are performed with considerable difficulty or are found to be well nigh impossible with the existing forms of electrical

[1] Brown, Phys. Rev., *4*, p. 85; 1914.
[2] Sieg and Brown, Phys. Rev., *4*, p. 507; 1914.
[3] Brown, Phys. Rev., *5*, p. 167; 1915.
[4] Brown and Sieg, Phil. Mag., *28*, p. 497; 1914.

measuring instruments. Among these is the accurate measurement of their exposure and recovery characteristics as dependent upon the wave length of the incident light. These data are readily obtained for x-ray and γ-ray frequencies[5] but in the region of visible radiation the changes on illumination occur too rapidly to permit the use of D'Arsonval or Thomson galvanometers of the essential sensitiveness. The experiments must here be adapted to the use of the Einthoven string galvanometer, adjusted to have a natural period of about 0.01 sec. It is well known that under such a condition the best string galvanometers are much less sensitive than the best instruments of the suspended coil or magnet type. Accordingly, several years ago, one of the writers set about to construct a crystal-bridge which would give a comparatively large change of current upon illumination, and thus render feasible many experiments utilizing the string galvanometer.

It was apparent that a sufficient number of very short crystal lengths connected in parallel would fulfill this purpose. The idea is a simple one depending merely upon long accepted laws of electrical resistance, but the path which finally led to the development of a successful bridge is fraught with several practical difficulties. The object of the present paper is to obviate these for future investigators in this field.

Two types of mounting have been found particularly good. Since they serve somewhat different purposes the construction of both will be described. In either case there are three essential parts to be considered, viz.:—1. A metallic grid (See Fig. 1); 2. A holder for the grid (Figs. 2 and 4); 3. A pressure device (Figs. 2, 3, and 4). The metallic grid serves to connect a large number of crystal lengths in parallel. In order to insure the stability of its position during use a grid holder is necessary. And, for the purpose of making good contacts between the numerous crystals and the gold fingers of the grid, a device for applying a uniform pressure over the effective area of the bridge must be provided.

In addition, an air-tight housing for the entire arrangement is advantageous in experiments of refinement where careful drying is advisable.[6]

THE GRID

Since the construction of the grid is practically the same for both types of mounting we will first discuss its manufacture. It is obvious

[5] McMahon, Phys. Rev., *16*, p. 558; 1920.
[6] Ries, Phys. Zeit, 9, p. 569; 1908.

that a thin film of metal deposted upon a smooth surface of glass or quartz and ruled as shown in Fig. 1 will serve the purpose of connecting a large number of short crystal lengths in parallel. As previously indicated, this is one of the principal features of the new bridges. Illumination of the crystals is effected through the transparent insulator. Quartz has been found to be considerably better than glass for the deposition of the metallic surface, although a good grade of glass, such as pyrex, does very well.

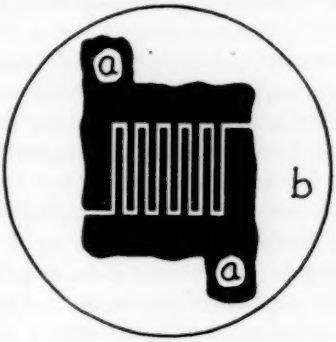


FIG. 1. *Diagram of grid.*
aa—Wood's metal connections to grid.
b—Plate of quartz or glass.

Several methods of depositing films of a number of different metals have been tried by the writers. Beautiful films may be secured, of course, by cathodic sputtering. But they do not adhere to glass and quartz firmly enough to insure the stability of the crystal contacts. Some success was obtained with gold leaf cemented to a glass plate with a very thin alcoholic solution of shellac, used sparingly. Good platinum films may be formed by the reduction of platinic chloride, but they are difficult to rule satisfactorily. The best grids thus far have been made from the gold and platinum paints[7] supplied for ceramic gilding by "Die Gold und Silber Scheide Anstalt" of Frankfort, Germany. Since the chemical action between gold and selenium is practically nil at ordinary temperatures[8] the gold grids are preferable.

Fortunately, the latter method is also the simplest. These gold and platinum paints consist of colloidal solutions of the metals in "essential oils." The writers find that thin applications of the paints, baked at

[7] The Devoe and Raynolds Co., New York City, handle these.
[8] Gmelin and Kraut, *3*, pp. 1014 and 1078, Anorganische Chemie.

a temperature somewhat lower than is customary with china painters, can be made to yield quite uniform films of the metals which will stand the close ruling desired in the grids for the crystal-bridge. According to Dr. Fritz Roessler, Director of the Frankfort laboratories, the films left after baking are composed of the pure metals.[9] This is important for the stability of the bridge. In fact, it may here be stated that the principal problems in the construction of a useful bridge lie in the elimination of the factors which produce electrical instability. It is for this reason that our method of making the grids will now be given in some detail.

The glass, or, preferably, quartz plate, which serves as the foundation for the crystal-bridge, must be plane, free from scratches and at least 2 mm thick. It is quite essential that even rather fine abrasions, visible to the naked eye by reflected light, be removed through the usual routine of polishing followed in optical shops. Otherwise, the ruling tool is likely to fail to remove the metal film from the scratch and the grid will prove to be worthless.

The plate is first treated with strong alkali and then with aqua regia to remove all grease and other impurities from its surface. It is washed with distilled water and finally with a good grade of ethyl alcohol. By means of an electric hot-plate it is now heated to 300° C for about thirty minutes. During the entire process the surface must be carefully protected from falling particles of dust. When the plate is cool again a thin film of the metallic paint is applied as indicated in Fig. 1. For this purpose a small camel's hair brush of best quality is cleared of dust and fine particles by washing in alcohol. All bubbles must be removed from the coating before heat is applied.

A more nearly uniform film is secured if the plate is now warmed fairly rapidly to about 250° C, without first allowing the paint to dry. This causes it to thin out considerably and flow over the surface somewhat, thus eliminating the thickened border which otherwise forms. The heating must be accomplished rapidly enough to bring about this initial flow, and yet not so fast that bubbles are formed. About 40 to 60 minutes of baking are required. (Care should be taken not to heat much longer since overheating renders the film too fragile for the purpose of ruling clean-cut lines of gold which adhere firmly to the plate under the conditions of the experiments for which the crystal-bridge is designed. On the other hand, the film must be heated long enough to drive off practically all of the "essential oils" used in the

[9] Beilby, Proc. Roy. Soc., London, *72*, p. 228.

manufacture of the metallic paints. When this is accomplished it is evidenced by the complete loss of the blackened appearance which the paint takes on in the early stages of the heating process.)

When the plate is cool the grid may be formed from the metallic film by means of a dividing engine. The spacing of the rulings is quite important to the stability and longevity of the bridge. After considerable experimentation the writers have found a spacing of two lines to the millimeter to be about the maximum for general purposes. The steel point used as a ruling tool has usually been sharpened to cut a line about 0.2 mm wide. This gives a width of 0.3 mm to the gold strips. We have made fairly successful bridges with more than twice as many rulings per millimeter, but they have been found to be less dependable over long periods of time. This is partially explained by the more extensive loosening of the metallic film during the ruling process in the latter case.

Satisfactory electrical connections may be made to the grid in several ways. Two small drops of Wood's metal, melted on as indicated in Fig. 1, have been found to serve very well for laboratory purposes. Where a more rugged construction is necessary the gold promontories may be plated lightly with copper and the connecting wires soldered. A good grid on quartz will show a resistance of over 100 megohms.

THE GRID HOLDERS

The principal requirement of the mounting is that it hold the grid plate firmly. Fig. 2, *C*, shows the brass mounting designed for a bridge using a screw type of pressure device. The plate is cemented to the ledge, *e*, in this mounting.

In the bridge represented in Fig. 4 the mounting has usually been a plate of thick glass large enough to accommodate a bell-jar placed over the entire arrangement. The quartz plate, which has formed the basis of all of the grids used in our scientific studies has been waxed on over a circular opening cut in the center of the larger glass plate. When in use, the latter plate has rested in a horizontal position upon four pads of sponge rubber, supported by a framework built to permit the illumination of the grid from below. With the exception of the quartz plate, these things are not shown in Fig. 4.

THE PRESSURE DEVICES

The two types of crystal-bridge here discussed are distinguished by the device used to apply the pressure necessary for good contact be-

tween the long needle-like crystals of selenium and the gold fingers of the grid. Fig. 2, *A*, illustrates in the barest outlines our screw type of pressure device. Fig. 3 shows an accessory spring and plate arrangement used with it. In Fig. 4 our gravity type is represented. The difference in purpose and relative advantages of the two bridges are

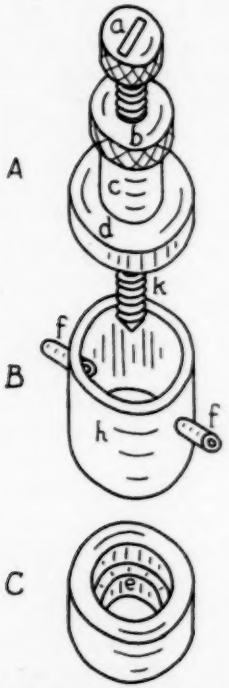


Fig. 2. *Outlines of brass parts for screw type of selenium crystal-bridge. Actual size: A—Screw type of pressure device; B—Cylindrical brass housing; C—Brass mount for grid plate. a—Knurled head for adjusting pressure screw. b—Lock nut. c—Matrix for screw. d—Base of pressure screw matrix. e—Ledge for supporting grid plate. ff—Ebonite plugs for insulating electrical connections. k—Pressure screw. h—Hollow brass cylinder.*

fairly apparent. The screw type is portable and may be used in any position. It is, therefore, a general utility bridge. But, electrically, it is not as stable as the gravity type. The latter is not portable and may be used with the grid in the horizontal position only. The gravity type of bridge is designed for careful measurements upon the electrical properties of selenium crystals where the highest degree of stability

is a prerequisite. Together, the two bridges cover a wide range of purposes.

In a large measure the figures are self-explanatory, but perhaps a word or two additional here and there will serve to emphasize several matters of importance.

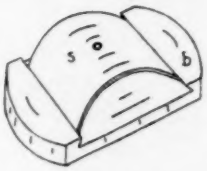


FIG. 3. *Spring and plate arrangement for screw type of pressure device. s—Thin piece of clock spring; b—Brass plate.*

The pitch of the screw, k, represented in Fig. 2, A, should be chosen as small as possible. We have found a pitch of 0.25 mm very satisfactory.

The parts A, B, and C, illustrated in Fig. 2, are fastened firmly together, in practice, by several long machine screws not shown in the diagram.

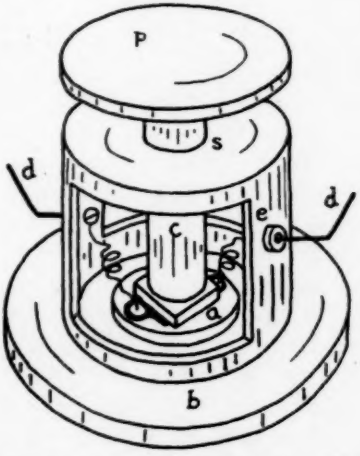


FIG. 4. *Gravity type of selenium crystal-bridge. One-half size. a—Grid plate, b—Brass frame, c—Cylinder Brass, dd—Electrical leads to the grid, e—Ebonite plug, p—Pan for weights, s—Sleeve.*

In Fig. 4 the details of the casing at s, through which the cylinder, c, operates, are not given. For the purpose of preventing side play this casing has been made three centimeters long and a close fit with the

cylinder obtained. A film of thin oil upon the cylinder renders the transmission of the pressure supplied by weights placed upon the pan, *p*, almost perfect.

The glass plate upon which the gravity type of bridge is mounted, together with its bell-jar cover and drying tube, are, for the purpose of simplification, not shown in Fig. 4.

THE SELECTION OF THE CRYSTALS

The production of the acicular crystals of selenium used in these bridges has been described elsewhere.[1] In order to secure the highest degree of stability the crystals should be approximately the same size and without such flaws as twisted structure, fractures, and promontories. They should be handled as little as possible, and then only with the smoothest forceps. These precautions are highly essential matters in the construction of a steady bridge. We have used crystals about 5 mm long and 0.2 to 0.3 mm thick in our bridges. Each one has been carefully examined on all sides with a jeweler's lens, and accepted only when known to be free from the above defects. About fifty crystals are required per square centimeter of grid area.

THE ASSEMBLY

The order and method of assembling the various parts will now be considered. Several steps in the process are vital and must be executed with care if the bridges are to show any permanence of value. During all of the construction one of the most important precautions is with regard to falling dust. Great vigilance must be exercised to keep both grid and crystals entirely free from foreign particles if the steadiest bridge possible is to be obtained.

In assembling the screw type of bridge we have usually placed the mounting, *C*, of Fig. 2, with the grid plate cemented to the ledge, *e*, upon a glass plate about 20 cm square resting in a horizontal position. The needle-like crystals are then layed on with their lengths perpendicular to the gold fingers of the grid. (We have not yet tried to construct bridges for the study of the electrical properties along the other crystalline axes.) They should not touch each other. The hollow cylinder *h*, is now fastened firmly to the mount and the leads from the grid brought out through the ebonite plugs, *ff*, and wedged tight. The grid and crystals are dried in situ by covering them with a bell-jar containing a vessel of pure phosphorus pentoxide. A small flat piece of a soft variety of paraffin, shaped to the outlines of the grid,

may be included in the drying chamber. After a few hours the bell-jar is removed and the final adjustments made as follows:

The paraffin slab is lowered carefully until it rests upon the crystals. It should cover the grid completely. The spring and plate arrangement shown in Fig. 3 is now placed upon the paraffin. It is centered as it is lowered, and should not be moved after it touches the paraffin. The base, *d*, of the pressure screw matrix, *c*, is screwed down upon the cylinder, *h*, and the pressure screw, *k*, adjusted until it just touches the center of the spring, *s*, in the small depression provided. The most important operation in the entire process of manufacture of this type of bridge—namely, the adjustment of the pressure to obtain the maximum sensitiveness and steadiness—must now be carried out. Increase of pressure increases the sensitiveness of the crystals to light over a considerable range.[1] But if the increase of pressure is taken beyond a certain point the bridge begins to lose the steady condition obtained after the first small application of pressure. To secure the best results the additional increments of pressure necessary should be made with the bridge connected to a battery of a few volts and a D'Arsonval galvanometer of sufficient sensitiveness. Time must be allowed for the flow of the paraffin between successive adjustments of the screw, *k*, which may be slowly advanced about a quarter of a turn at a time. Sometimes even less is advisable. As soon as the galvanometer gives a little jump the motion of the screw should be stopped. The adjustment is complete when the paraffin touches all around the edges of the grid. The nut, *b*, is provided for locking the pressure screw. Subsequently, it is usually necessary to take up the flow of the paraffin occasionally until it has settled better. (In all of these adjustments we may well note that an error of too much pressure practically always ruins the steadiness of the bridge. An explanation is given in a later paragraph.)

The final steps in the construction of the gravity type of bridge must be carried out in the position in which it is to be used. When the crystals have been deposited upon the grid, the base of the brass frame, *b*, with the cylinder, *c*, and pad, shown in Fig. 4, removed, is lowered upon the large glass plate carrying the quartz plate, *a*, where it is centered and waxed. Electrical connections are made as indicated in the drawing. It is convenient to take the leads, *dd*, out through the glass plate mentioned above but not shown in the figure. In this bridge the paraffin slab used in the screw type of bridge is replaced by a rubber pad. The cylinder, *c*, is now gently lowered to rest upon the pad. For each square centimeter of grid area about 1 kg is applied to the pan, *p*

(where the arrangement of the crystals is that previously given). The entire bridge is now covered with a bell-jar which is waxed tightly to the large glass plate. A side tube carrying a little pure phosphorus pentoxide is provided to keep the grid and crystals dry.

SENSITIVENESS AND STEADINESS

A representative specimen of the screw type of bridge possessed the following characteristics one month after its construction and adjustment:—

Applied Potential	Resistance in the Dark	Resistance with 500 millilumens per sq. cm.
2.0 Volts	18,000 Ohms	6,800 Ohms
20.0 "	14,000 "	6,700 "

We see, therefore, that a current change as high as 1.56 milliamperes may be secured with this bridge in rather weak light. (For constant use the voltage should be kept low.)

The gravity type of bridge was developed for scientific purposes where the steadiest electrical behavior is the prime requisite. It is easier to maintain a constant resistance by this method than by the use of a screw device, since the resistance is so sensitive to pressure changes. The irregular variations in the dark current through the screw type of bridge are of the order of 10^{-7} amperes under favorable conditions, while in the gravity type with the more constant pressure the irregularities are from ten to a hundred times smaller.

A THEORY OF THE UNSTEADINESS

During the past ten years progress in the study of the fascinating light-sensitive properties of selenium crystals has been considerably hindered by a phenomenon of unsteadiness familiar to all who have worked with them. In a large measure we have eliminated this difficulty in addition to developing the grid feature of the bridges, which brings several hundred times the current change given by a single crystal on illumination. In working out our process of constructing the bridges we have been guided by the following view of the causes of the unsteadiness of the resistance in the dark.

When we began the investigation of the difficulty it was soon discovered that there was a large residuum of unsteadiness after the crystals were thoroughly dried and the conditions for good contacts complied with. It should be noted that in all of our experiments variations due to air currents and temperature changes have been eliminated. With these precautions we entered upon a more careful study of what

happens as the pressure is increased. It was found that the steadiness becomes a maximum at a rather small value shortly after the increase is begun. In other words, with no pressure upon the crystals, the behavior of the electrical resistance of the bridge is somewhat erratic. As the pressure is increased marked improvement is noted for a short interval. But after a certain point any further augmentation of the pressure results in an increasing condition of unsteadiness until the bridge is rendered unfit for use. It is important to note that relieving the crystals and reapplying a smaller pressure does not restore the steadiness.

These facts have led us to the following explanation of the phenomenon. The initial erratic behavior of the bridge is probably due to the mechanical instability of the contacts. As the pressure is increased this is eliminated. Improvement is noted as long as the forces applied are in the interval over which Hooke's law is obeyed. But when their elastic limit is reached the crystals begin to fracture and flow upon a microscopic scale. If this process goes on erratically, as it likely does, the phenomenon of the unsteady electrical resistance in the dark is explained. Obviously, releasing and reapplying the pressure would not remedy the fractures.

The actual flow of the crystals when the bridge is in the second unsteady region has been amply verified. When the crystals of a screw type of bridge, which had been kept in the above condition for several weeks, were examined, it was found that they had flattened out completely, losing all semblance of their hexagonal form except on a microscopic scale.

All of these things, we believe, show that the selenium crystals obey Hooke's law over a range of forces which is quite limited. Steady electrical conditions can be obtained only within this range.

In conclusion, we take pleasure in acknowledging our indebtedness to the Department of Physics of the University of Iowa for facilities placed at our disposal during the early part of these investigations, made in 1917. In 1922 the studies were continued in the Physical Laboratory of the Rice Institute. We wish to thank the Trustees of the Institute for their very kind grant of the funds with which the work was carried on.

THE RICE INSTITUTE, (A. M. M.)
HOUSTON, TEXAS.
BUREAU OF STANDARDS. (F. C. B.)
WASHINGTON, D. C.

Necessary Conditions for Combination of colliding Atoms into Molecules.—When two free atoms collide and join into a molecule, what becomes of their translational kinetic energies (initial, plus what they acquire as they are drawn together)? Some will pass into translational kinetic energy of the resulting molecule, but not in general all, for the value of this is prescribed by the principle of conservation of momentum, and will, in general, not be exactly equal to the sum of the initial energies. The residue might pass into internal energy of the molecule, but here we meet the difficulty that the molecule should admit only certain specific values of internal energy to be determined by quantum-conditions, and in general none of these will be of just the right amount to take up the available energy. Or the residue might be radiated; but from the correspondence principle Born and Franck infer that this is impossible unless the atoms are initially charged. The alternative is to assert that the atoms cannot combine unless a third strikes them while they are in collision, or unless they both strike a solid; this may be the general explanation for catalysis. The authors make a number of speculations awaiting verification; for example, that atoms may collide and fly apart again within 10^{-12} second, and yet in this short period of time constitute a sort of molecule able to absorb and to emit light of frequencies more or less nearly equal to the frequencies due to electron-transitions in stable molecules, without, however, the perfectly definite modifications due to quantized rotations and to quantized vibrations in stable molecules.—[M. Born and J. Franck; Ann. d. Phys. *76*, pp. 225-230, 1925; ZS. f. Phys. *31*, pp. 411-429, 1925].

KARL K. DARROW

Colors of thin Films on Metals.—To prove that the colors of thin films on metals are due to interference between beams of light reflected from their outer and inner surfaces, and not to a postulated granular structure, Evans varies the thickness of films produced in certain ways and finds a concomitant variation in color such as the former theory requires. In one case he dissolves off, by an electrolytic process, a portion of a film produced by heating iron in a flame ("temper colors"); in another he observes films formed on the surface of lead melted in air, which may be skimmed off and examined in transmitted light as well as by reflected, giving complementary instead of identical colors in the two modes of inspection. In the latter case the film may be liquid and yet colored, which seems to dispose of the idea that the color is due to granular structure. An analysis of other published and unpublished work leads Evans to the same conclusion.—[U. R. Evans; Proc. Roy. Soc. *A107*, pp. 228-237, 1925].

KARL K. DARROW

ANALYSIS OF BI-METAL THERMOSTATS

By S. Timoshenko

INTRODUCTION

The following investigation contains a general theory of bending of a bi-metal strip submitted to a uniform heating. This theory is applied in analysis of operation of a bi-metal strip thermostat. The equations are obtained for calculating the temperature of buckling, the complete travel during buckling, and the temperature of buckling in a backward direction. By using these equations the dimensions of the thermostat for a given temperature of operation and a given complete range of temperature can be calculated. The results obtained are based on certain ideal conditions. For example, it was assumed that the difference in the coefficients of expansion remained constant during heating, that the friction at the supports could be neglected and that the width

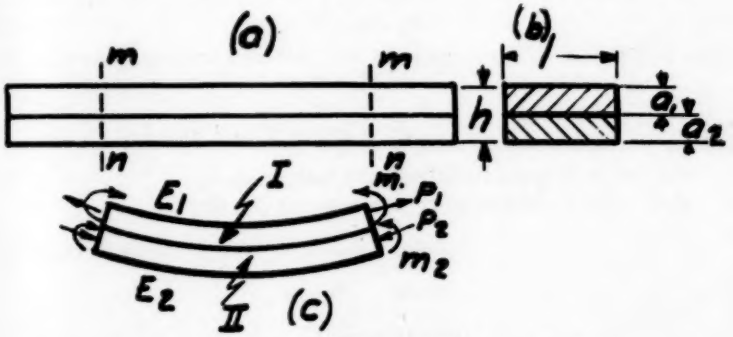



Fig. 1. *Deflection of a bi-metal strip while uniformly heated.*

of the strip could be considered as being very small. It is the opinion that these assumptions are not far removed from real conditions and the results obtained by using this theory can be considered as a useful guide in choosing the dimensions of thermostats and in discussing the stresses occurring in bi-metallic strips during operation.

1. *Deflection of a bi-metal strip while uniformly heated.* Let a narrow strip consisting of two metals welded together be uniformly heated from $t_0°$ to $t°$C. If the coefficient of linear expansion of these metals be different the heating will produce bending of the strip.

Let α_1 and α_2 denote the coefficients of expansion of the two metals (1) and (2), (see Fig. 1).

E_1 and E_2 denote their moduli of elasticity,
a_1 and a_2 their thicknesses,
h the thickness of the strip.

The width of the strip is taken as equal to unity. The following analysis has been made on the assumption that cross-sections of the strip originally plane and perpendicular to the axis remain plane during bending and become perpendicular to the curved axis of the strip. Consider an element cut out from the strip by two cross-sections mn and m_1n_1. If $a_2 > a_1$, the deflection will be convex down, as shown in Fig. 1 (c). All the forces acting over the section of metal (1) on the concave side can be represented by an axial tensile force P_1 and bending moment M_1. For metal (2) on the convex side all forces acting on the cross-section can be represented by an axial compressive force P_2 and bending moment M_2. Due to the fact that there are not external forces acting on the strip, all forces acting over any cross-section of the strip must be in equilibrium, therefore,

$$P_1 = P_2 = P \tag{1}$$

$$\frac{Ph}{2} = M_1 + M_2 \tag{2}$$

Letting ρ = radius of curvature of strip,
E_1I_1 = the flexural rigidity of the metal (1)
E_2I_2 = the flexural rigidity of the metal (2), then,

$$M_1 = \frac{E_1I_1}{\rho}$$

$$M_2 = \frac{E_2I_2}{\rho}$$

Substituting in (2)

$$\frac{Ph}{2} = \frac{E_1I_1 + E_2I_2}{\rho} \tag{3}$$

Another equation for calculating P and ρ will be obtained from the consideration of deformation. On the bearing surface of both metals the unit elongation occurring in the longitudinal fibres of metals (1) and (2) must be equal, therefore,

$$a_1(t - t_0) + \frac{P_1}{E_1a_1} + \frac{a_1}{2\rho} = a_2(t - t_0) - \frac{P_2}{E_2a_2} - \frac{a_2}{2\rho} \quad \text{or}$$

using (1) and (3)

$$\frac{h}{2\rho}+\frac{2(E_1I_1+E_2I_2)}{h\rho}\left(\frac{1}{E_1a_1}+\frac{1}{E_2a_2}\right)=(a_2-a_1)\,(t-t_0)$$

From which,

$$\frac{1}{\rho}=\frac{(a_2-a_1)\,(t-t_0)}{\frac{h}{2}+\frac{2(E_1I_1+E_2I_2)}{h}\left(\frac{1}{E_1a_1}+\frac{1}{E_2a_2}\right)}$$

Letting,

$$\frac{a_1}{a_2}=m\,,\qquad \frac{E_1}{E_2}=n\,,$$

and remembering that

$$I_1=\frac{a_1^3}{12}\,,\qquad I_2=\frac{a_2^3}{12}$$

the following general equation for the curvature of a bi-metal strip will be obtained,

$$\frac{1}{\rho}=\frac{6(a_2-a_1)\,(t-t_0)\,(1+m)^2}{h\left(3(1+m)^2+(1+mn)\left(m^2+\frac{1}{mn}\right)\right)} \tag{4}$$

If the thicknesses of both metals are equal,

$$a_1=a_2\,,\quad m=1$$

$$\frac{1}{\rho}=\frac{24(a_2-a_1)\,(t-t_0)}{h(14+n+1/n)} \tag{5}$$

The curvature is proportional to the difference in elongation of the two metals and inversely proportional to the thickness of the strip. It is seen that the magnitude of the ratio $\frac{E_1}{E_2}=n$ does not produce any substantial effect on the curvature of the strip. Take for instance, $n=1$, then,

$$\frac{1}{\rho}=\tfrac{3}{2}\frac{(a_2-a_1)\,(t-t_0)}{h} \tag{6}$$

For $n=1/2$ or $n=2$,

$$\frac{1}{\rho}=\frac{48}{33}\frac{(a_2-a_1)(t-t_0)}{h} \tag{7}$$

The difference between (6) and (7) is only about 3 per cent.

When the curvature $\frac{1}{\rho}$ is obtained the deflection δ of the strip can be easily evaluated. For instance, let a strip AB of bi-metal simply supported at the ends be bent due to uniform heating, as shown in Fig. 2. Then from the fact that the curve ACB is an arc of a circle having radius ρ it follows, that, $\delta(2\rho-\delta)=(l/2)^2$. Or remembering that the deflection δ is small in comparison with the radius of curvature we have,

$$2\rho\delta=\frac{l^2}{4},$$

from which,

$$\delta=\frac{l^2}{8\rho} \tag{8}$$

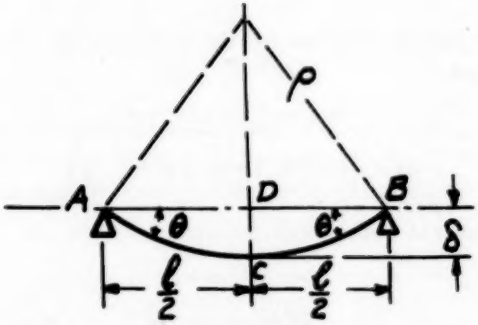


FIG. 2. *Deflection of a simply supported bi-metal strip.*

2. *Internal stresses during heating of a bi-metal strip.* The maximum stress during heating of bi-metal will be produced in the fibres on the bearing surface of the two metals. This stress consists of two parts: (a) the stress due to axial force and (b) the stress due to bending. For metal (1) it will be determined by the equation,

$$p_{max}=\frac{P}{a}+\frac{a_1E_1}{2\rho}$$

or by using (3)

$$p_{max}=\frac{1}{\rho}\left(\frac{2}{ha_1}(E_1I_1+E_2I_2)+\frac{a_1E_1}{2}\right)$$

In the simplest case,

$$E_1 = E_2 = E$$

$$a_1 = a_2 = \tfrac{1}{2}h$$

$$p_{max} = \frac{E}{\rho}\,\frac{h}{4}(\tfrac{1}{2}+1)\text{ , or using }(b)\text{ ,}$$

$$p_{max} = \frac{E}{2}(\alpha_2 - \alpha_1)(t - t_0) \qquad (9)$$

Letting $E = 27\times10^6$ lbs./sq. in.

$$\alpha_2 - \alpha_1 = 4\times10^{-6}$$

$$t - t_0 = 200°C$$

the following is obtained,

$$p_{max} = 10800 \text{ lbs./sq. in.}$$

The stress distribution over the cross-section of the strip is shown in Fig. 3. It is easy to see that all stresses distributed over the cross-section are in equilibrium. They can be reduced to two equal couples of opposite directions.

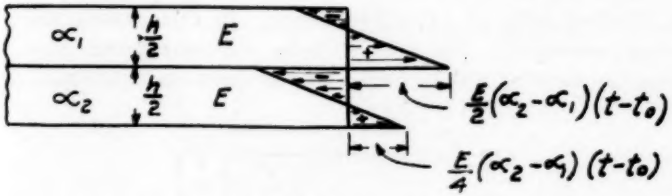


FIG. 3. *Distribution of normal stresses on any cross section of bi-metal strip while uniformly heated.*

In all previous discussion cross-sections remote from the ends of the strip were considered. Near the ends the picture of stress-distribution is more complicated. In the simplest case of two metals having equal thicknesses and equal moduli, shearing stresses between the two metals near the end will be produced during heating, as shown in Fig. 4. For every material these stresses can be reduced to a central force P, used in previous calculations, and to a couple $\frac{Ph}{4}$. The stresses produced by these forces are shown in Fig. 3 above. The distribution of shearing stress along the bearing surface cannot be determined in an elementary

way and it can be stated only that they are of "local" type and are concentrated near the ends of the strip along a distance, the magnitude of which is of the same order as the thickness h of the strip. This shows that the maximum intensity of these stresses may be of the same order as that of the normal stresses calculated above. This conclusion may be of some importance in considering the elastic property of material used for welding the two metals.

FIG. 4. *Shearing stresses acting near the ends of bi-metal strip.*

If the modulii and thicknesses of the two metals are different, not only sheering stresses but also normal stresses along the bearing surface near the ends of the strip will be produced during heating. These latter stresses will be also of local character and can be reduced to two equal and opposite couples of such magnitude as is necessary to produce, together with couples $\frac{Pa_1}{4}$ and $\frac{Pa_2}{4}$, previously considered, equal curvatures $1/\rho$ in both metals.

3. *Bending produced by external forces.* In this consideration the previous assumption that cross-sections, originally plane, remain so during the bending will be used. The problem on bending of a bar

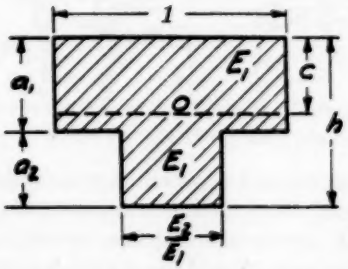


FIG. 5. *Equivalent section of homogeneous strip.*

consisting of two materials welded together can then be solved as follows.

Assume that $E_2 < E_1$ (Fig. 1). This fact can be taken into account by considering the bending of a strip of homogeneous material but of a section as shown in Fig. 5. The diminishing of the width corresponding

to the less rigid material must be taken in the ratio $\frac{E_1}{E_2}$ if the flexibility of strips has to remain the same. The stress distribution due to bending will depend on the position of the neutral axis going through the center of gravity O of the section. Let c determine the position of the center of gravity, then by using previous notation,

$$\frac{E_1}{E_2} = n\,, \quad \frac{a_1}{a_2} = m$$

$$c = \frac{\frac{(a_1+a_2)^2}{2} + \left(1-\frac{1}{n}\right)\frac{a_1^2}{2}}{a_1+\frac{a_2}{n}}$$

In the case, $a_1 = a_2 = h/2$

$$c = \frac{h(1+1/4\,(n-1))}{n+1} \tag{10}$$

The moment of inertia of the cross-section about the neutral axis will be,

$$I = \frac{(a_1+a_2)^3}{3n} + (1-1/n)\frac{a_1^3}{3} - \left(a_1 + \frac{a_2}{n}\right)c^2$$

In the case, $a_1 = a_2 = 1/2\ h$,

$$I = h^3\left(\frac{7}{24n} + \frac{1}{24} - \frac{\left(\frac{3}{n}+1\right)^2}{32(1+1/n)}\right) \tag{11}$$

If both materials have the same modulus $n = 1$ and from (11), we have

$$I = h^3\left(\tfrac{1}{3}-\tfrac{1}{4}\right) = \frac{h^3}{12} = .0833h^3$$

For $n = 1/.85$

$$I = h^3(.2896 - .2128) = .0768h^3 \tag{12}$$

For $n = 2$

$$I = .0573\ h^3 \tag{13}$$

It is interesting to compare these results with the results obtained for the flexural rigidity of the strip, considering it as homogeneous and taking for the modulus of elasticity the average value $E = 1/2\ (E_1+E_2)$. Then,

$$EI = \frac{h^3E_1}{12} \times \tfrac{1}{2}\left(1 + \frac{E_2}{E_1}\right) = \frac{h^3E_1}{12} \times \frac{1+n}{2n} \tag{14}$$

For $n = 1/.85$, equation (14) gives

$$EI = \frac{E_1 h^3}{1} \times \frac{1.85}{2} = .0772\, E_1 h^3$$

This is in very good agreement with the equation (12).
For $n = 2$,

$$EI = \frac{h^3 E_1}{12} \times \tfrac{3}{4} = .0625\, E_1 h^3$$

As compared with (13) it gives a difference of about 9 per cent.

From this it can be concluded that in the calculation of flexural rigidity of a strip of bi-metal the formula for a homogeneous bar can be used with an average value of the modulus of elasticity. If n is near to unity (in the case of monel metal and nickel steel $n = 1/.85$) this method of calculation is accurate enough.

By using this conclusion, the stresses produced in a constrained bi-metallic strip during heating can be easily obtained. Consider first

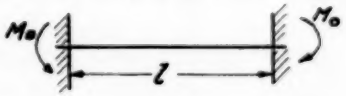


FIG. 6. *Bi-metal strip with clamped ends.*

a bi-metallic strip with built-in ends (Fig. 6). During uniform heating couples M_0 will occur at the ends. The magnitude of these couples must be such as to annihilate the curvature $1/\rho$ produced by heating (see Eq. 6). The equation for calculating M_0 becomes,

$$\frac{M_0}{EI} = 1/\rho = \tfrac{3}{2} \frac{(\alpha_2 - \alpha_1)(t - t_0)}{h}$$

In which,

$$E = \frac{E_1 + E_2}{2}$$

$$I = \frac{h^3}{12}$$

The corresponding maximum bending stress,

$$p = \frac{6 M_0}{h^2} = \frac{6}{h^2} \times \tfrac{3}{2} EI \frac{(\alpha_2 - \alpha_1)(t - t_0)}{h} = \tfrac{3}{4} E(\alpha_2 - \alpha_1)(t - t_0)$$

Combining the bending stresses due to M_0 with those produced in a free bi-metallic strip Fig. 3, the stress distribution shown in Fig. 7 will be obtained.

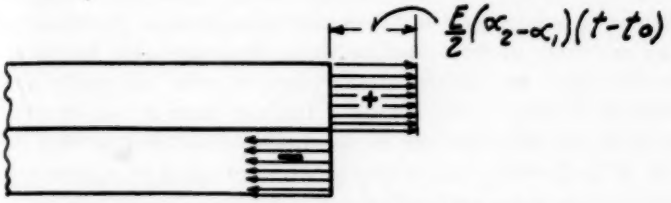


FIG. 7. *Stress distribution in bi-metal strip with clamped ends.*

In the case of a bi-metallic strip built in at the end A and having a pin at the end B (Fig. 8) the reactive force R must be such as to annihilate the deflection δ which would be produced by heating of a strip

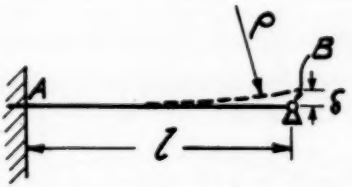


FIG. 8. *Bending of bi-metal strip with one end clamped and another simply supported.*

free at the end B. By using eq. 6 and the deflection formula for a cantilever beam, we have,

$$\frac{l^2}{2\rho} = \frac{Rl^3}{3EI}$$

from which,

$$R = \tfrac{3}{2}\frac{EI}{l} \times 1/\rho = \tfrac{9}{4}\frac{EI}{hl}(\alpha_2-\alpha_1)\,(t-t_0)$$

The maximum bending moment at the built in end becomes

$$M_{max} = Rl = \tfrac{9}{4}\frac{EI}{h}(\alpha_2-\alpha_1)\,(t-t_0)$$

and the corresponding maximum bending stress,

$$p_{max} = \frac{6\,M_{max}}{h^2} = \tfrac{9}{8}\,E(\alpha_2-\alpha_1)\,(t-t_0)$$

This stress is 50 per cent higher than that for the strip having both ends built in, which was obtained above.

4. *Deflection of a Bi-Metal Plate while uniformly heated.* If instead of a narrow strip a plate of bi-metal be heated, the curvatures in two different directions must be taken into consideration. For small deflections and with uniform heating these curvatures will be equal and constant over the surface of the plate, so that the plate becomes spherical in form. The radius of the curvature ρ can be obtained exactly in the same manner as shown above for the case of a narrow strip. It is necessary only to take into consideration that layers parallel to the neutral surface will suffer, due to spherical bending equal tension or compression in two perpendicular directions. Due to this fact, the equations of paragraph (1) can be used also in this case. It is necessary only to substitute $\frac{E_1}{1-\sigma}$ and $\frac{E_2}{1-\sigma}$ instead of E_1 and E_2 where σ denotes Poisson's ratio considered as equal for both materials. Due to the fact that the final equations (4) and (6) contain the ratio $\frac{E_2}{E_1}$ only, they will remain without any change and can be used in calculating the curvature during bending of the plates.

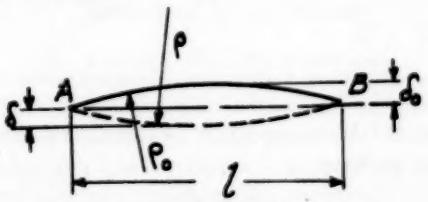


FIG. 9. *Bending of curved bi-metal strip.*

It is clear that the bending of the plate into a spherical form is impossible without some strain in the middle surface of the plate. If the deflections are small, say less than a half of the thickness of the plate, this strain in the middle surface of the plate can be neglected without any essential error, but with the increasing of the deflection, the effect of this strain becomes more and more pronounced and must be taken into consideration.

B. ANALYSIS OF THE STRIP TYPE OF BI-METAL THERMOSTAT

5. *Free bending of bi-metal strip.* Let a strip of bi-metal have an initial curvature equal to $1/\rho_0$ an initial deflection δ_0 (Fig. 9) and be bent in such a manner that the material with the larger coefficient of

expansion is on the concave side. During heating of this strip the curvature will gradually diminish and at a certain temperature the strip will pass through the plane form and afterwards become curved in a downward direction as shown by the dotted line in Fig. 9. The variation of the curvature will be obtained from equation (6)

$$\frac{1}{\rho_0}+\frac{1}{\rho}=\frac{3}{2}\frac{(a_2-a_1)\,(t-t_0)}{h} \tag{15}$$

The corresponding deflection from (8) and (15) will be

$$\delta=\frac{l^2}{8\rho}=\frac{l^2}{8}\left(\frac{3}{2}\frac{(a_2-a_1)\,(t-t_0)}{h}-\frac{1}{\rho_0}\right)=\frac{3}{16}\frac{l^2}{h}(a_2-a_1)\,(t-t_0)-\delta_0 \tag{16}$$

For a thermostat a sudden buckling instead of gradual variation of curvature is essential and in order to obtain this characteristic fastening of the ends of the strip is necessary.

6. *Bi-metal curved strip in a rigid frame.* We will consider here the simplest case of a thermostat in the form of a curved bi-metal strip fastened between the fixed supports A and B in a rigid frame (see Fig. 10). It is assumed that the temperature expansions of the frame and

FIG. 10. *Bi-metal curved strip in a rigid frame.*

center line of the strip are equal and that there is no pressure of the strip on the supports at the initial temperature. Let l = length of the strip. The width is taken as equal to unity.

ρ_0 = the initial radius of curvature.

δ_0 = the initial deflection of the strip.

$A = 1 \times h$ = the area of the cross section of the strip.

$I=\dfrac{1\times h^3}{12}$ = the moment of inertia of the cross section.

$E=\frac{1}{2}(E_1+E_2)$ = the average modulus of elasticity.

δ = the deflection in a downward direction of the strip after heating for the case of "free ends" (see eq. 16).

δ_1 = the deflection measured in upward direction at which a sudden buckling of the strip begins.

δ_2 = the deflection of the strip in downward direction after buckling.
t_0 = the initial temperature of the strip.
t = the temperature at which sudden buckling occurs.
t_1 = the temperature when buckling in backward direction takes place.
$\Delta t = t - t_1$ = the range of the temperature.
t_2 = the temperature when the deflection for the case of "free ends" is equal to δ_0.

The deformation of the strip during heating may be divided into two stages: first, a gradual decrease of the deflection from the initial value δ_0 to some value δ_1, and secondly, a sudden buckling in a downward direction as shown by the dotted line in Fig. 10. During the gradual decrease of the initial curvature of the strip a longitudinal compression of the strip will occur. The corresponding compressive forces P tend to oppose buckling and may prevent it, provided the increase in temperature be not sufficiently great.

The temperature of buckling and the deflection δ_1 from which sudden buckling begins may be found if the following two expressions have been determined:

1. An expression giving the force P as a function of the decrease of the deflection of the strip during heating.

2. The magnitude of the compressive forces P necessary to prevent buckling of the strip when the temperature and the deflection are both given.

In order to obtain the increase in the compressive force due to decrease in the deflection, assume that the center line of the bent strip is a flat curve. Then the difference λ between the length of the curve and the length of the chord will be

$$\lambda = \tfrac{1}{2} \int_0^l \left(\frac{dy}{dx}\right)^2 dx \tag{17}$$

Without any great error it can be assumed that the initial deflection curve of the strip is a sine curve. Then we have,

$$y = \delta_0 \sin \frac{\pi x}{l}$$

Substituting in (17)

$$\lambda = \frac{\delta_0^2 \pi^2}{4l} \tag{18}$$

When, during heating, the deflection of the strip diminishes from δ_0 to δ_1, the corresponding compression of the strip, from eq. (18), becomes,

$$\frac{\delta_0^2\pi^2}{4l} - \frac{\delta_1^2\pi^2}{4l}$$

and the compressive force will be,

$$P = \left(\frac{\delta_0^2\pi^2}{4l} - \frac{\delta_1^2\pi^2}{4l}\right)\frac{EA}{l} = \frac{EA\,\pi^2}{4l^2}\delta_0^2\left(1 - \frac{\delta_1^2}{\delta_0^2}\right) \quad \text{or}$$

$$\frac{P\,l^2}{E\,I\pi^2} = \frac{3\delta_0^2}{h^2}\left(1 - \frac{\delta_1^2}{\delta_0^2}\right) \tag{19}$$

This equation gives the force P as a function of the ratio $\frac{\delta_1}{\delta_0}$.

In evaluating the force necessary to prevent buckling of the strip, assume first that the supports A and B (see Fig. 10) are removed and the strip is free to assume during heating the form indicated by the line ACB in Fig. 11. The corresponding deflection δ may be calculated from equation (16). Assume now that this strip is bent in a direction convex up as shown by the dotted line and is held in this bent form by com-

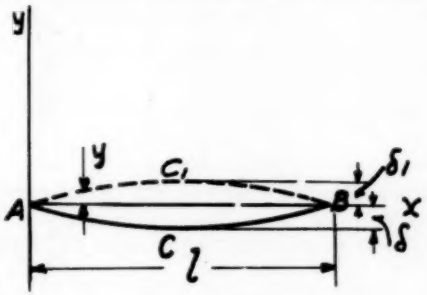


FIG. 11. *Deflection curve of bi-metal strip.*

pressive forces P. It is clear that the magnitude of the forces P will depend on the magnitude of δ_1. In order to obtain the relation between δ_1 and P, assume that the initial curve ACB (Fig. 11) is a sine curve

$$\delta \sin\frac{\pi x}{l}$$

Then the displacements in the y direction occurring during bending from the shape ACB to the shape AC_1B are

$$y + \delta \sin\frac{\pi x}{l}$$

and the differential equation of the deflection curve AC_1B will be

$$EI\frac{d^2}{dx^2}\left(y+\delta\sin\frac{\pi x}{l}\right)=-Py$$

from which,

$$\frac{d^2y}{dx^2}+\frac{P}{EI}y=\frac{\delta\pi^2}{l^2}\sin\frac{\pi x}{l} \tag{20}$$

Using notation,

$$\frac{P}{EI}=k^2 \tag{21}$$

we have,

$$y=A\sin kx+B\cos kx+\frac{\delta}{\dfrac{k^2l^2}{\pi^2}-1}\sin\frac{\pi x}{l} \tag{22}$$

where A and B are arbitrary constants of integration. Excluding the case when $\frac{kl}{\pi}$ = an integer* it can be concluded from the conditions at the ends that $A=B=O$.

Therefore,

$$\delta_1=\frac{\delta}{\dfrac{k^2l^2}{\pi^2}-1}$$

or

$$\frac{Pl^2}{EI\pi^2}=\frac{\delta}{\delta_1}+1 \tag{23}$$

Substituting for δ expression (16)

$$\frac{Pl^2}{EI\pi^2}=\left(\frac{3}{16}\,\frac{l^2}{h\delta_0}(a_2-a_1)\,(t-t_0)-1\right)\frac{\delta_0}{\delta_1}+1 \tag{24}$$

This equation gives the force P necessary to hold the strip ACB in the convex upwards shape AC_1B, shown by the dotted line in Fig. 11. It is seen that the magnitude of P increases with the decrease of the deflection δ_1.

Equations (24) and (19) give the complete solution of the problem on buckling of the strip. If the force evaluated from (24) is always larger than that given by equation (19) this means that the reactions P at the supports A and B (Fig. 10) are insufficient to prevent buckling. On the other hand, if for a certain value of the ratio $\frac{\delta_0}{\delta_1}$ the magnitude

* The corresponding values of P are given by the know Euler's column formula.

of P, as calculated from (24), becomes less than that obtained from equation (19), the reactions P of the frame will prevent buckling of the strip.

7. *Graphical Solution.* In order to determine the limit of temperature increase $t-t_0$, which is just necessary to produce the buckling of the strip, the following graphical method may be used.

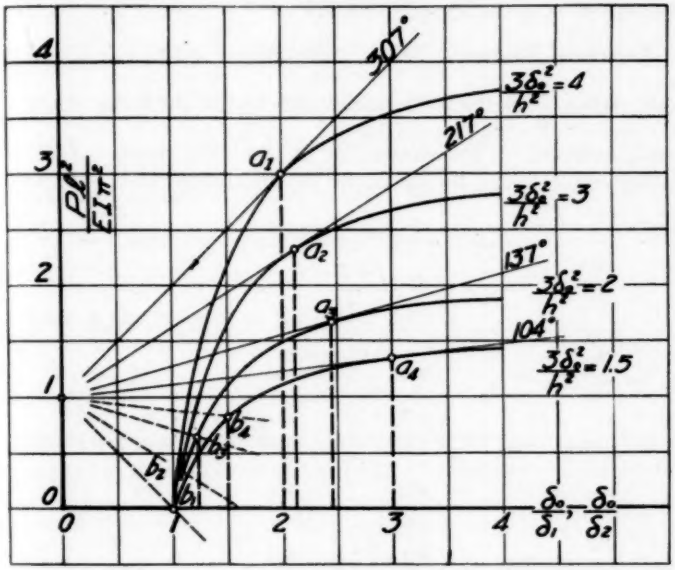


FIG. 12. *Graphical determination of the temperature of operation of a bi-metal strip thermostat.*

By applying equation (19), the curves representing $\frac{Pl^2}{EI\pi^2}$ as a function of $\frac{\delta_0}{\delta_1}$ are first plotted. Several curves of this kind, corresponding to various values of the ratio $\frac{3\delta_0^2}{h^2}$ are shown in Fig. 12. These curves have as their asymptotes horizontal lines distant $\frac{3\delta_0^2}{h^2}$ from the abscissa axis.

Furthermore, equation (24) may be represented graphically by straight lines going through the point *1* (Fig. 12) having the coordinates $x=0$, $y=\frac{Pl^2}{EI\pi^2}=1$. The slopes of these lines are given by the equation,

$$i=\tfrac{3}{16}\frac{l^2}{h\delta_0}(\alpha_2-\alpha_1)(t-t_0)-1 \qquad (25)$$

It is seen that the slope depends on the temperature increase $t-t_0$. If the temperature increase is small, i also will be small and the straight line will cut the corresponding curve, given by equation (19). It means that at such a temperature no buckling occurs. The limiting temperature increase at which buckling becomes possible will be determined by the value of the slope of the tangent drawn through A to the corresponding curve, given by equation (19).

8. *Numerical Example.* Take, for instance, $\frac{h}{l}=100$; $\alpha_2-\alpha_1=4\times 10^{-6}$ and the ratios $\frac{\delta_0}{h}$ of the initial deflection to the thickness of the strip are such as given by the second line of the table above. The corresponding curves given by equation (19) are plotted as previously dis-

TABLE A

$\frac{3\delta_0^2}{h^2}=1.5$	2	3	4
$\frac{\delta_0}{h}=.707$	.814	1.000	1.154
$i=.111$	.272	.628	1.00
$t-t_0=104°$	137°	217°	307°C
$\frac{\delta_1}{\delta_0}=.333$	.408	.472	.500
$\frac{\delta_2}{\delta_0}=.667$	.813	.940	1
$t_1-t_0=83°$	79°	50°	0°
$\Delta t=21°$	58°	167°	307°

cussed. The values of the slopes of the tangents drawn through the point A to these curves are given in the third line of the table. Substituting these values in equation (25) the temperatures of buckling are obtained. These are given in the fourth line of the table.

It is seen that with increase of the initial deflection δ_0, the temperature of operation increases. The slightest change in δ_0 produces a large effect on the temperature of buckling.

The abscissae of the points of tangency a_1, a_2, a_3 and a_4 (see Fig. 12) represent the values of the ratio $\frac{\delta_0}{\delta_1}$ at which sudden buckling of the strips commences. The corresponding values of $\frac{\delta_1}{\delta_0}$ are given in the fifth line of the table.

9. *Analytical Solution.* The same solution can be found also in an analytical way. By using notations,

$$\frac{Pl^2}{EI\pi^2} = y$$

$$\frac{\delta_0}{\delta_1} = x$$

$$\frac{3\delta_0^2}{h^2} = a$$

$$\frac{3}{16}\frac{l^2}{h\delta_0}(a_2 - a_1)(t - t_0) - 1 = i$$

we have, from equations (10) and (24),

$$y = a(1 - 1/x^2) \qquad (19)^1$$

$$y = ix + 1 \qquad (24)^1$$

The slope of the tangent to the curve $(19)^1$ is

$$\frac{dy}{dx} = \frac{2a}{x^3} \qquad (26)$$

Substituting in $(24)^1$ expressions (26) and $(19)^1$ for i and y, the following expression for the abscissae of the point of tangency will be obtained,

$$x_0 = \frac{\delta_0}{\delta_1} = \sqrt{\frac{3a}{a-1}} \qquad (27)$$

Substituting in this equation $a = 1.5$, 2, 3, and 4, the reciprocal of the numbers given in the fifth line of the table above will be obtained. The slope of the tangent, from (26) and (27), becomes,

$$i = 2a(1/3 - 1/3a)^{3/2} \qquad (28)$$

Substituting this in equation (25) we have

$$t - t_0 = \frac{1 + \frac{6\delta_0^2}{h^2}\left(\frac{1}{3} - \frac{1}{9}\frac{h^2}{\delta_0^2}\right)^{3/2}}{\frac{3}{16}\frac{l^2}{h\delta_0}(a_2 - a_1)} \qquad (29)$$

From this equation the temperature of operation can be evaluated.

10. *Deflection of the strip after sudden buckling.* From the analysis given above it is seen that during heating of the strip the deflection,

initially equal to δ_0, gradually decreases. When it attains the magnitude δ_1, given by equation (27), a sudden buckling occurs and the strip, initially convex up (Fig. 10), becomes convex down. The differential equation of the deflection curve after buckling will be the same as equation (22) but the deflection will be now in downwards direction and its magnitude δ_2 will be given by the equation,

$$\delta_2 = \frac{\delta}{1 - \frac{kl^2}{\pi^2}}$$

Instead of equations (23) and (24) we will have

$$\frac{kl^2}{\pi^2} = \frac{Pl^2}{EI\pi^2} = 1 - \frac{\delta}{\delta_2} \tag{23}^1$$

$$\frac{Pl^2}{EI\pi^2} = 1 - \left(\frac{3}{16}\frac{l^2}{h\delta_0}(a_2 - a_1)\,(t - t_0) - 1\right)\frac{\delta_0}{\delta_2} \tag{24}^1$$

Instead of equation (19) we will have,

$$\frac{Pl^2}{EI\pi^2} = \frac{3\delta_0^2}{h^2}\left(1 - \frac{\delta_2^2}{\delta_0^2}\right) \tag{19}^1$$

A very simple graphical solution of equations $(19)^1$ and $(24)^1$ is shown in Fig. 12. In order to obtain the magnitude of $\frac{\delta_0}{\delta_2}$ it is only necessary to draw through point *1* the lines $1b_1$, $1b_2$ . . . . having slopes,

$$-i = -\left(\frac{3}{16}\frac{l^2}{h\delta_0}(a_2 - a_1)\,(t - t_0) - 1\right)$$

Then the abscissae of points b_1, b_2 . . . . give the values of $\frac{\delta_0}{\delta_2}$ and the ordinates, the corresponding values of $\frac{Pl^2}{EI\pi^2}$. This solution enables us to calculate very easily the displacements of the strip during sudden buckling. Take, for instance, the numerical example considered above and assume $\frac{3\delta_0^2}{h^2} = 2$. Then from Fig. 12 and Table *A*, it is seen that the sudden buckling begins when,

$$\delta_1 = .408\,\delta_0$$

The deflection in a downwards direction after buckling is equal to $\delta_1 = .813\delta_0$. The complete displacement during sudden buckling is,

$$\delta_1 + \delta_2 = 1.22\,\delta_0$$

11. *Temperature at which Reverse Buckling Occurs.* If after sudden buckling of the strip the cooling begins, a certain limit t_1 for the lowering of the temperature may be established at which the strip becomes unstable and a sudden buckling in a backwards direction occurs so that the temperature controlled by the thermostat will vary within limits from t to t_1. The range of temperatures $\Delta t = t - t_1$ is of practical importance and can be easily evaluated as follows.

Let t_2 denote the temperature necessary to produce in the strip with "free ends" the deflection δ equal to δ_0 (see equation (16)). At such a temperature the buckled strip will not press against the supports A and B of the frame, i.e., the conditions will be the same as in the initial unbuckled state. But now the material with the higher coefficient of expansion is on the convex side. Due to this fact the cooling will produce on the buckled strip the same effect as heating of the strip in its initial shape. The lowering of temperature $t_2 - t_1$ necessary to produce buckling in a backwards direction will be equal to the raising of the temperature $t - t_0$, calculated above (see table A). Therefore the temperature t_1 of the action of the thermostat in the backwards direction will be found from the equation,

$$t_2 - t_1 = t - t_0$$

from which

$$t_1 = t_2 - (t - t_0) \tag{30}$$

Here t_2 is determined from the equation,

$$\frac{3}{16}\frac{l^2}{h\delta_0}(\alpha_2 - \alpha_1)(t_2 - t_0) = 2 \tag{31}$$

By using equations (30) and (31) the numbers of the last two lines in the table A were obtained. It is seen from this table that with the lowering of the temperature of action the range Δt of temperatures diminishes, i.e., the thermostat becomes more sensitive to temperature variation.

From (25) and (31)

$$(t_2 - t_0) = \frac{2}{1+i}(t - t_0)$$

Substituting in (30)

$$t_1 - t_0 = \frac{1-i}{1+i}(t-t_0) \tag{32}$$

In order to obtain large sensitiveness the quantity i must be taken very small, i.e., $\frac{3\delta_0^2}{h^2}$ must be near to unity and δ_0 near to .577 h. Practically, it is impossible to have sufficient accuracy with such a small initial deflection and in order to obtain sufficient sensitiveness with larger initial deflection the employment of a flexible frame becomes necessary.

12. *Bi-metal Curved Strip in a Flexible Frame.* Assume that due to the flexibility of the frame the reactive forces P (see Fig. 10) produce an increase Δ in the distance between the supports A and B. Then the compression of the strip produced by these forces during heating will be

$$\frac{\delta_0^2\pi^2}{4l} - \frac{\delta_1^2\pi^2}{4l} - \Delta$$

and the corresponding compressive force,

$$P = \left(\frac{\delta_0^2\pi^2}{4l} - \frac{\delta_1^2\pi^2}{4l} - \Delta\right)\frac{EA}{l} \tag{a}$$

In order to obtain the magnitude of Δ the flexibility of the frame must be taken into consideration. Assume that this flexibility is equivalent to the extensibility of a strip having the length l and the cross section A_1. Then,

$$\Delta = \frac{Pl}{EA_1}$$

Substituting in (a)

$$P = k \cdot \frac{EA\pi^2}{4l^2}\delta_0^2\left(1 - \frac{\delta_1^2}{\delta_0^2}\right) \qquad \text{where,}$$

$$k = \frac{A_1}{A+A_1} \tag{33}$$

Instead of equation (19) we will have

$$\frac{Pl^2}{EI\pi^2} = k \cdot \frac{3\delta_0^2}{h^2}\left(1 - \frac{\delta_1^2}{\delta_0^2}\right) \tag{34}$$

Equation (24) remains without change. Now from (34) and (24) the temperature of operation can be obtained exactly in the same manner as explained above in the case of a rigid frame. The same graphical

method as shown in Fig. 12 can be used; it is necessary only to take the ordinates of the curves equal to

$$k\frac{3\delta_0^2}{h^2} \text{ instead of } \frac{3\delta_0^2}{h^2}$$

Take, for example, $k=1/6$ and $\delta_0=2\ h$, then,

$$k\frac{3\delta_0^2}{h^2}=2$$

The slope of the line Aa_3 in Fig. 12 will determine the temperature of operation. The magnitude of the slope, from Table A, is equal to .272 and equation (25) becomes

$$\frac{3}{16}\frac{l^2}{h\delta_0}(\alpha_2-\alpha_1)\,(t-t_0)=1.272$$

Substituting the previous data, we have,

$$t-t_0=338°\text{C}$$

The temperature of backward buckling, from equation (32), becomes

$$t_1-t_0=\frac{1-i}{1+i}\left(t-t_0\right)\frac{1-.272}{1+.272}\times 338=193°C\ .$$

It is easy to see that a greater sensitiveness may be obtained by increasing the flexibility of the frame, i.e., by decreasing the slope i.

Applying equations (24), (32) and (34) the dimensions of the strip can be evaluated in such a manner so as to obtain, theoretically speaking, any given sensitiveness and any given temperature of operation. By using notations of the paragraph (8) these equations become,

$$\frac{t_1-t_0}{t-t_0}=\frac{1-i}{1+i} \qquad (32)^1$$

$$y=ka\left(1-\frac{1}{x^2}\right) \qquad (19)^{11}$$

$$y=ix+1 \qquad (24)^{11}$$

From (19),^{11}

$$\frac{dy}{dx}=\frac{2ka}{x^3} \qquad (a)$$

Substituting (a) and $(19)^{11}$ in $(24)^{11}$

$$ka\left(1-\frac{1}{x^2}\right)=\frac{2ka}{x^2}+1$$

from which,

$$x_0 = \frac{\delta_0}{\delta_1} = \sqrt{\frac{3\,ka}{ka-1}} \tag{35}$$

Substituting in (a)

$$i = 2ka(\tfrac{1}{3} - 1/3ka)^{3/2} \tag{36}$$

Therefore,

$$t - t_0 = \frac{1 + 2ka(\frac{1}{3} - 1/3ka)^{3/2}}{\frac{l^2}{\frac{3}{16}h\delta_0}(a_2 - a_1)} \tag{37}$$

Assume, for instance, that the thermostat must have a temperature of operation $t - t_0 = 300°$ and have such a sensitiveness that the backward buckling occurs when the lowering of the temperature becomes equal to 10 per cent of the temperature of operation, then, from (32)[1]

$$\frac{1-i}{1+i} = .9 \text{ or } i = .053$$

Substituting in (36)

$$ka = k\frac{3\delta_0^2}{h^2} = 1.29 \tag{b}$$

Now, from (37),

$$\frac{l^2}{h\delta_0} = \frac{1+i}{\frac{3}{16}(t-t_0)(a_2-a_1)} = \frac{1.053}{\frac{3}{16}\times 300\times 4\times 10^{-6}} = \frac{4\times 1.053\times 10^4}{9}$$

Taking, for instance,

$$l/h = 200\text{, we have,}$$

$$\delta_0 = 9/1.053h = 8.55h$$

From (35) we have,

$$\frac{\delta_0}{\delta_1} = 3.65 \tag{b}$$

From equations,

$$y = -ix - 1$$

$$y = ka(1 - 1/x^2)$$

determining the deflection δ_2 of the strip after bending, we have,

$$\frac{\delta_0}{\delta_2} = 1.70 \qquad (c)$$

The range of the complete travel during operation is

$$\delta_1 + \delta_2 = \delta_0\left(\frac{1}{3.65} + \frac{1}{1.70}\right) = .86\delta_0 \qquad (d)$$

It is seen that by increasing the ratio l/h the initial deflection and the range of displacement (d) can be increased.

SCIENTIFIC PAPER 178,
WESTINGHOUSE RESEARCH LABORATORY.
EAST PITTSBURGH, PA.

Refraktion und Akkommodation des menschlichen Auges, Professor Dr. A. Siegrist, Director of the Eye Clinic in the University of Bern; pp. vi+148, with 108 illustrations, mostly colored. (Julius Springer, Berlin, 1925).

This exceedingly useful and attractive little volume is the outgrowth of a course of lectures on Refraction and Accommodation which Professor Siegrist has been in the habit of giving in the University of Bern. Apparently, therefore, the book is intended primarily for medical students in ophthalmology. The author is careful to state in the preface that he has taken particular pains to avoid all mathematical formulae; and so very little previous knowledge even of elementary geometrical optics is required in order to read the book. Nowadays when spectacle optics and methods of refraction are being developed in so many ways, one wonders if a skillful ophthalmologist can continue to be ignorant of such mathematics and physics as are almost necessary to gain a real insight into the intricate problems of the dioptrics of the eye and to ascertain the precise effect of using a lens or prism for the correction of the so-called anomalies of vision.

However, this book is evidently written by a practical ophthalmologist of wide experience and contains much valuable and authoritative information. Besides, it is beautifully illustrated, most of the pictures being colored. Some of them are of very great special interest. In the first ten pages there is an excellent modern description of the anatomy of the eye. This is followed by a chapter on the various types of spectacle glasses. Here the writer gives also an account of attempts that have been made to devise thin shell glasses to fit over the cornea and correct the curvature of this surface when it is very irregular. He describes also several forms of "hydrodiascope" for the same purpose, one of which he invented himself. The effect of accommodation on the intraocular pressure is discussed in the chapter on accommodation. The objections to Snellen's test charts for determining visual acuity are very clearly set forth, and various modern tests are described, especi-

ally those of Pflüger, Landolt, Sulzer and v. Hess. Few ophthalmologists or optometrists in this country or in Europe seem to be familiar with Ives's visual acuity test object, which has some great merits in the reviewer's opinion.

The latter part of the volume (pages 75-132) is, perhaps, the most interesting and original, where the writer treats very clearly the so-called "errors" of refraction (hypermetropia, myopia, astigmatism), ophthalmoscopic appearances, the etiology of the different kinds of refraction and the prophylaxis of myopia. Much valuable information will be found here in connection with all these subjects. Incidentally, the ordinary reader will find a practical description of the clinical form of ophthalmometer and the way it is used for measuring corneal astigmatism.

On the other hand, the various methods of refraction and optometry are not included within the scope of this treatise. The ocular muscles are mentioned only incidentally, if at all.

It is an admirable book for its purpose, and should be of the greatest value to students and practitioners.

J. P. C. SOUTHALL

Refinements of the method of measuring resonance potentials.—The essence of this contribution is contained in this paragraph: "Superpose upon the potential V, between the filament and grid (of an ordinary 3-electrode or 4-electrode tube), a small alternating potential dV of frequency n, so that the applied voltage oscillates between the values $V+dV$ and $V-dV$. The current to the plate will accordingly osciliate between the values $I+dI$ and $I-dI$. Let now a commutator rotating with the frequency n be placed in the plate circuit. The current, after passing through the commutator, will oscillate between $dI+I$ and $dI-I$; that is, it will consist of an alternating part I, and a unidirectional part dI. Any galvanometer whose period is large compared with $1/n$ will give a steady reading proportional to dI, so that if we keep dV constant and vary V, the galvanometer will approximately record the differential of the I vs. V curve when dV is small." Sample curves presented in the article actually show a much more striking singularity in the neighborhood of the resonance potential than do the curves obtained in the usual manner, suggesting that the method might prove valuable. The variable potential may also be applied, together with the commutator, in the plate-grid circuit. The applications made by the author are rather difficult to follow in his account, and the results thus far attained are apparently not sharply definite, owing to the complexities of the distribution-in-velocity of the electrons and the dependence of the probability of inelastic impact upon speed of the electron; he hopes to learn something about the latter if the former can be properly settled.—[E. G. Dymond, Cavendish; Proc. Roy. Soc. *A107*, pp. 291-309, 1925.]

KARL K. DARROW

THE THEORY OF THE RADIOMETER

By H. E. Marsh, E. Condon, and L. B. Loeb

The theory of the radiometer has been successfully developed by Knudsen[1] for pressures so low that the mean free path of the gas molecules is comparable with the dimensions of the vane and enclosing vessel. In spite of this practically nothing is known about the theory of the vane radiometer, often seen in opticians' windows, which operates at much higher pressures. Gerlach and Westphal[2,4,5] have carried on a series of excellent experimental investigations in recent years without, however, having materially extended the theory published by Westphal[3] in 1919. Recently two papers on the higher pressure radiometers have appeared. One was by Hettner and Czerny,[6] and the other by A. Einstein.[7] The former deals with the radiometric effects which are caused by molecular convection currents set up along the surfaces of the vanes by local temperature gradients. These effects explain the action of the repulsion radiometer of the Nichols type, and of certain other peculiar forms. In the latter article A. Einstein[7] discusses the action of the ordinary vane type of radiometer on the basis of very elementary kinetic theory, and his conclusions differ quite radically from those resulting from the earlier theory of Westphal.[3] His argument is essentially as follows. In a free gas space in which there exists a temperature gradient one can imagine a plane drawn normal to the gradient. Through such a plane there is no molecular streaming, (i.e. no convection currents exist), while at the same time heat transfer takes place due to the difference in momentum of the molecules on the two sides of the plane. Under these circumstances an actual difference of pressure would manifest itself on two sides of a particle whose dimensions are small compared to the mean free path, thus causing it to move in the direction of the lower temperature. On the other hand it is well known that equality of pressure would exist on the two sides of a large material vane whose dimensions are great compared to the mean free path of the

[1] Knudsen, Ann. d. Physik, *32*, p. 809; 1910.
[2] Gerlach and Westphal, Verh. d. D. Phys. Ges. *21*, p. 218; 1919.
[3] Westphal, Verh. d. D. Phys. Ges. *21*, p. 129; 1919.
[4] Westphal, ZS. für Phys. *1*, p. 92; 1920; ZS. für Phys. *4*, p. 221; 1921.
[5] Gerlach, ZS. für Phys., *2*, p. 207; 1920.
[6] Hettner and Czerny, ZS. für Phys., *27*, p. 12; 1924.
[7] A. Einstein, ZS. für Phys. *27*, p. 1; 1924.

gas molecules if this vane were placed in a region occupied by the imaginary vane. Thus it appears, according to Einstein, that, for the case of a large vane in a large gas space in which there is a gradient of temperature, there must exist a transitional region at the edge of the vane in which the condition of no resultant pressure for the solid vane changes to one of a resultant pressure in the free gas space. According to this theory therefore the region of the width of a mean free path along the edge of the vane is the seat of all the radiometric effects of this type.[8] For the case of a temperature gradient through an imaginary plane vane, Einstein arrives at a resultant force per unit length of edge given by the following equation:

$$K = -\frac{1}{2} p\lambda^2 T \frac{\partial T}{\partial x}$$

For the case of the ordinary radiometer vane, blackened on one side and having a temperature difference ΔT in the direction of its thickness due to illumination on one face, Einstein gives an equation, (applicable in order of magnitude only) which takes the form,

$$K = -p\lambda \frac{\Delta T}{T}$$

where K is the force on the vane per unit length of edge, p is the gas pressure, ΔT is the difference in temperature, λ is the mean free path, T the absolute temperature and $\partial T/\partial x$ is the temperature gradient through the gas in the first equation.

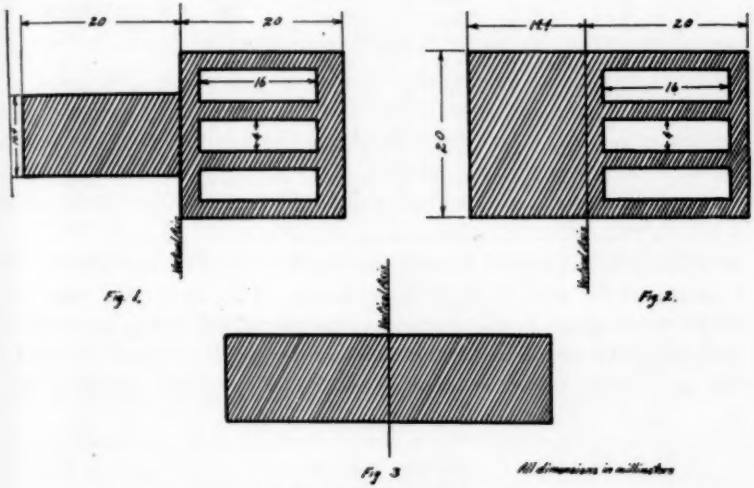
If this theory is correct the radiometric pressure should be proportional to the length of the edge rather than to the area of the vane as hitherto supposed. It was with a view to testing this conclusion from the theory that the authors undertook the preliminary experiments which are here reported.

Several specimens of two types of a thin mica vane, (shown in Figs. 1 and 2) were first constructed. These vanes were 0.045 mm in thickness. They were so designed that, when suspended to rotate about the line marked "vertical axis," they would have the same area moments about this line but the edge moments would be in the ratio of approximately 4 for the grid to 1 for the solid side. In design 1 the areas were equal as well as the area moments. They were blackened on one side with lamp black and suspended by means of a fine quartz fibre in a brass frame which was so arranged that the vanes with their suspensions could be

[8] G. D. West, Prac. Phys. Soc., London, *32* p. 172, 1920, states "On a Crookes vane it is supposed that the excess of pressure is restricted to a narrow margin near the edge." The theory on which this is based differs from that given above.

easily interchanged. The frame was placed in a bell jar 25 cm in diameter which could be evacuated to pressures of 0.005 mm of mercury. The pressures were read on a McLeod gauge.

By means of a lens and a diaphragm a parallel beam of light from a 500 watt locomotive headlight could be thrown so as to illuminate the vane alone. This beam was tested for homogeneity with a thermopile and only that portion used which was sufficiently homogeneous. The deflections of the vane were registered by a beam of light on a small mirror fastened immediately above the vane. By means of a movable



Illustrating designs of radiometer vanes used.

diaphragm four sets of readings could be taken as follows: (1) with all the light cut off for a zero reading, (2) with the entire vane illuminated, (3) with the grid alone illuminated, and (4) with the solid end alone illuminated. In general it was found that the difference between the deflections for the grid alone illuminated and with the solid end alone illuminated was equal to the deflection with the whole vane illuminated. Readings* taken with the thin mica vanes of designs 1 and 2, and with pressures of about 0.05 mm, are given in table 1.

As a control, a third design was constructed as shown in Fig. 3. This vane was perfectly symmetrical about the vertical axis, 0.045 mm thick,

* In these readings the position of the frame supporting the suspension and vane relative to the portion of the vane illuminated had no effect on the result obtained.

blackened on one side, and suspended as before. The deflections are shown in Table 2.

From the above data it can be seen at once that the greater edge moment has the advantage, the deflections being towards the grid side at all times. The ratio of the deflections with the grid to the deflections

TABLE 1.

Design	Press. in mm	Thin Mica Vanes, Designs 1 and 2. (Thickness, 0.045 mm)			Total vane illuminated. (Deflection to Grid side)
		Grid Illum.	Solid Illum.	Diff.	
1	0.05	7.0	5.8	1.2	1.4
1	"	10.8	5.6	5.2	4.7
1	"	7.4	5.8	1.6	1.6
1	"	17.2	12.3	4.9	4.3
2	"	12.7	7.7	5.0	4.2
2	0.08	22.2	7.4	14.8	16.6
2	0.20	4.8	3.9	0.9	0.8
2	0.08	6.1	5.7	0.4	0.4

with the solid end should however be approximately in the ratio of 4 to 1, as this is the ratio of the edge moments. This, it is seen, does not hold even roughly. It was suspected that the failure of this law to hold quantitatively was due to another factor. As the equations show, the radiometer pressures depend on the temperature difference on

TABLE 2.

Pressure	Thin Mica Vane, Design 3. (Thickness 0.045 mm)			Total
	Right Side Illum.	Left Side Illum.	Diff.	
0.05 mm	0.5	0.5	0	0

the two sides, as well as on the edge. If this factor could be made the same for the two parts of the vane the effect should be in the quantitatively correct ratio. However, (as Hettner points out) in the case of such vanes the edges produce a cooling. Thus the grid has a smaller ΔT than the solid end which accounts for the observed reduction in the ratio of the observed deflections.

To test this matter further a thin aluminum vane, 0.15 mm thick and of type 1, was constructed and blackened on one side. The results are shown in Table 3. It is seen that in this case the deflections are all greater towards the solid end than towards the grid, and that with both

sides illuminated the vane still shows a deflection towards the solid end; the deflection with both illuminated however not being equal to the difference in deflection of the separate ends. It was anticipated that this result was due to a conduction of heat from one side of the vane to

TABLE 3.

Press. in mm	Grid Illum.	Aluminum Vane, Design 1.		Total vane illuminated. Deflection to solid side.
		Solid Illum.	Diff.	
0.10	17.00	18.0	1.0	6.3
0.05	10.7	26.0	15.3	22.0

the other, thus producing temperature differences on the non-illuminated portions of the vanes. This assumption was substantiated by cutting the vane in two along its axis and separating the two halves by a thin glass rod. From Table 4 it will be seen that the metal vane, under these conditions, behaved very much like the thin mica vanes.

TABLE 4.

Press. in mm	Grid Illum.	Aluminum Vane, Two Halves Insulated.		Total vane illuminated. Deflection to grid side.
		Solid Illum.	Diff.	
0.10	1.7	1.0	0.7	0.4
0.19	1.5	1.3	0.2	0.2
0.07	5.8	4.5	1.3	1.4

A further test of the cooling effect of the edges was made by means of a thick mica vane in which one might expect a relatively smaller cooling effect at the edges. Such a vane should also have a greater temperature difference on the two sides than the thinner vanes. Accordingly a vane of design 2, 0.45 mm thick, was constructed. The deflections are shown in Table 5. It is seen here that the grid end is twice as effective as the solid end in producing deflections, i.e., the ratio of the deflections of this vane approached more nearly the theoretical value expected. As was expected the greater ΔT caused a marked increase in the sensitivity of the vanes which was more than five times as sensitive as the thinner ones. The deflections recorded were made with greatly reduced illumination.

The sensitivity was found to increase as the pressure decreased. The maximum sensitivity was not reached in the experiments recorded.

This variation of sensitivity with pressure is not in accord with the second equation of Einstein which is supposed to be applicable to this case, for the equation demands that the force vary as λp which is independent of the pressure. However, the value of ΔT is also likely to be decreased at the higher pressures, due to increased conduction, so that complete independence of pressure is not predicted by the equation quoted. However, such an effect would not be nearly as great as that observed.* The beautiful observations of Gerlach[3] and of Westphal[4] on the variation of the radiometric effect with pressure are in accord

TABLE 5.

Pressure in mm	Thick Mica Vane, Design 2. (Thickness 0.45 mm)			Total vane illuminated. Deflection to Grid side.
	Grid Illum.	Solid Illum.	Diff.	
0.05	15.6	7.6	8.0	8.1
"	3.9	1.8	2.1	2.1
"	4.0	2.0	2.0	2.1
"	17.1	7.8	9.9	7.8

with the observations of the writers. The first equation of Einstein, applying to the ideal case, however, leads one to expect an increase in sensitivity with pressure decrease until pressures are reached where the mean free path becomes comparable with the dimensions of the vane. At this point a whole new type of radiometric activity (the Knudsen type) begins.

Thus the theory applicable to this type of vane as given in the Einstein article, is distinctly contrary to the experimental pressure observations and, until agreement in this respect occurs, the theory can hardly be said to be verified. However, this investigation has revealed one thing very definitely and that is that *it seems to be the edge of the vanes rather than the area which plays the important part in the action of this type of radiometer*. Further experiments, by one of us, are in progress to determine the importance of this effect and to throw some light on the theory from this point of view.

PHYSICS DEPARTMENT,
UNIVERSITY OF CALIFORNIA,
BERKELEY, CAL.
FEBRUARY 25, 1925.

* Later results show that the change in ΔT with pressure is probably ample to cause the pressure variation observed.

STORAGE BATTERIES*

By G. W. Vinal

Storage batteries find many uses in the laboratory where the requirements are often most exacting. A summary of the characteristics and operation of the batteries may, therefore, be helpful. A brief description of the essential parts of the cells and the theory of operation is followed by a discussion of the factors affecting capacity, the characteristics of charge and discharge, resistance, efficiency and sources of trouble. Alkaline cells are also described. Short bibliography.

CONTENTS

I. Lead-acid cells 263
1. The plates 263
2. The Separators 264
3. The Electrolyte 264
4. Reactions and Voltage 266
5. Factors affecting capacity 267
6. Characteristics of charge and discharge 268
7. Resistance 269
8. Efficiency 270
9. Sources of Trouble 270
II. Alkaline Storage cells 272

Secondary cells are generally called storage cells or accumulators. They convert chemical energy into electrical energy by reactions that are essentially reversible; that is, they may be charged by an electric current passing through them in the opposite direction to that of their discharge. Two types are of commercial importance, the lead-acid cells and the nickel-iron or alkaline cells. The positive terminal is defined as that from which the current flows to the external circuit when the battery discharges.

I LEAD-ACID CELLS

(1) *The plates.* The active material of the positive plates is lead peroxide (dark brown), and of the negative plates sponge lead (grey). When these materials are formed from the underlying lead of the plates, they are referred to as "Planté plates." The active materials of the "pasted" plates are formed by electrolytic oxidation (positives) and reduction (negatives) of lead oxides which are made into a paste

* Published by permission of the Director of the Bureau of Standards of the U. S. Department of Commerce.

One of a series of reports prepared at the request of the Committee on Research Methods and Technique appointed by the Division of Physical Sciences of the National Research Council. It is the purpose of these reports to present in brief outline a summary, more or less critical, of the methods which may be employed in various kinds of Physical measurements. See Editorial in this Journal *9*, p. 410, 1924.

with dilute solutions of sulphuric acid or of ammonium sulphate. The paste is supported by a cast alloy grid of lead with about 7 per cent antimony. The grid also serves to conduct most of the current to and from the active material. The active material of both plates is porous and permits the electrolyte to diffuse through the plates as they discharge or are charged. This is necessary for the proper operation of the cell. The sponge lead of the negative plates contains a small amount of an "expander" such as lamp-black, barium sulphate or wood flour to prevent the lead sponge from shrinking and losing its porosity. The positive plates even when fully charged contain about 3 per cent of lead sulphate which can not be easily removed by excess charging. This sulphate serves an important rôle as a binder for the lead peroxide, which has little coherence.

(2) *The separators.* The primary object of the separators is to prevent metallic conduction between plates of opposite polarity while freely permitting electrolytic conduction. Separators are generally made of wood such as bass, poplar, cedar, fir, cypress or redwood. The separators are treated to remove soluble and easily hydrolyzed matter and to expand the pores. For reasons not fully understood, the presence of wood in the battery is beneficial and necessary for the proper functioning of the negative plates. Some negative plates, therefore, contain wood. The corrugated side of the separator is placed next to the positive plates because they require more acid than the negatives. Perforated rubber sheets are often placed between the positive plates and the separators. These assist in holding the lead peroxide in place and decrease the oxidizing effect of the peroxide on the wood. However, they increase the resistance of the cell. Threaded rubber separators consist of thin sheets of rubber containing several hundred thousand threads passing from one side to the other like small wicks.

(3) *The electrolyte.* A solution of pure sulphuric acid in pure water should be used for batteries of the lead-acid type. The proper strength of the solution, which is usually expressed by the specific gravity, varies somewhat with the kind of battery and the conditions of service. In the storage battery industry the word "electrolyte" is used primarily to mean the solution within the cells. When space is limited a more concentrated solution must be used and the range of the specific gravity from the charged to the discharged condition will be greater than for a battery containing a larger volume of solution of moderate concentration. Local action at the negative plates and the attack of the acid on

the separators limit the maximum specific gravity for any type to about 1.300. The proper specific gravities of the solution for different kinds of batteries when fully charged are approximately as follows:

Aviation batteries	1.300 sp. gr.
Automobile, tractor, and vehicle batteries	1.280 to 1.300
Signal batteries	1.220 to 1.250
Stationary batteries	1.210
Radio batteries, A circuit	1.250
Radio batteries, B circuit	1.225

The local action of negative plates in 1.300 specific gravity solutions is nearly 2.5 times as great as in solutions of 1.200 specific gravity. Local action is greatly increased by a rise in temperature. At 30°C the effect is double that at 22°C.

A high degree of purity is necessary. Impurities such as platinum, silver, and copper which are intermediate in potential between the lead and lead-peroxide, deposit on the negative plate and produce local action, but they are not all equally bad. Those with the highest potential differences and the lowest hydrogen over-voltages make the most trouble. Iron is successively oxidized to the ferric condition at the positive plate and reduced to the ferrous salt at the negative, and so it discharges the battery without depositing on either plate. Manganese is thrown out as the dioxide in the pores of the positive plate and interferes with the charging of these plates. Bismuth, antimony, and arsenic affect the negative plates. Nitric acid is a solvent for lead and attacks the negatives. Chlorine attacks both the positive and negative plates. Part of it is oxidized to perchloric acid and remains in the solution. Acetic acid produces less effect than is generally supposed. It is gradually decomposed during charging and escapes as carbon dioxide. Organic impurities, such as separator extracts, sucrose and starch affect the positive plates.

Distilled water is preferred for flushing the batteries. Natural water varies in purity from place to place and from one time of the year to another. When natural water is used the impurities which it contains are cumulative in the battery. Clean rain water or melted snow can generally be used satisfactorily. Acid should not be added to the cells except to replace that which has been spilled or otherwise lost.

Many attempts have been made to improve the electrolyte by adding various substances to it, but few, if any, have merit and none of them

are necessary for the fundamental reactions which occur in the battery. Jelly electrolytes, made from sodium silicate and sulphuric acid, decrease the capacity and life of the battery. The battery is not charged by changing the solution as is sometimes claimed; any additional capacity that may be obtained is at the expense of residual active materials already present in the battery.

(4) *Reactions and voltage.* There is abundant evidence that the so-called double sulphate theory of the reactions is the most probable. The significance of the term double sulphate lies in the fact that lead sulphate is formed at both positive and negative plates during discharge. It is most conveniently stated by the equation:

$$PbO_2+Pb+2H_2SO_4\rightleftarrows 2PbSO_4+2H_2O$$

From left to right this equation represents discharge and from right to left charge. The equation does not tell how the substances are formed at the plates and their relation to the transfer of electricity through the cell. LeBlanc's theory of the tetravalent and divalent lead ions is perhaps the most probable (Lehrbuch der Elektrochemie, p. 223, 1895; Storage Batteries by Vinal p. 149, 1924). The heat of the reaction in the cell depends on the heats of formation of the various substances taking part in the reaction. For an ordinary cell containing 2 H_2SO_4. 18 H_2O the heat of the reaction on discharge is somewhat more than 90,000 calories, which represents heat energy transformed into electrical energy and the Gibbs-Helmholtz equation for such a cell may be written

$$\frac{dE}{dT}=\frac{1}{T}(E-0.000021674Q)$$

in which E is the open circuit voltage, T the absolute temperature, and Q the heat of the reaction. Observed and calculated values for $\frac{dE}{dT}$ are in good agreement, about 0.0004 volt per degree C (Vinal and Altrup, Scientific Paper, Bureau of Standards No. 434). There is a small reversible heat effect of about 5000 calories. This heat is absorbed during discharge and liberated during charge. It should not be confused with the irreversible heat effect arising from the ohmic resistance. The voltage of the cell varies with the concentration of the acid in the pores; for this reason a cell on discharge behaves as if it were in a more dilute solution than the main body of electrolyte while on charge it behaves as if in a more concentrated electrolyte. The open circuit voltage varies with the concentration as follows:

Specific Gravity	Volts
1.040	1.890
1.100	1.956
1.200	2.045
1.300	2.138
1.400	2.233
1.83	2.60

(5) *Factors affecting capacity.* The capacity is usually stated as the ampere-hour capacity, but the watt-hour capacity may be easily calculated by multiplying the ampere-hour capacity by the average voltage during discharge. Batteries may be rated for ampere-hour capacity in several ways, but the most satisfactory is the time rating by which the capacity is stated to be a certain number of ampere-hours which can be delivered in a specified time such as 8 hours, 5 hours, or 20 minutes. When rated on the time basis, the capacity is proportional to the number of positive plates per cell.

The entire theoretical capacity of a battery can not be obtained because the electrolyte does not diffuse into the pores with sufficient rapidity when the pores are clogged with lead sulphate and the resistance of the active material and of the electrolyte increases rapidly near the end of the discharge. An arbitrary end-point is therefore chosen and specified as a closed-circuit voltage below which it is not desirable to discharge the battery. This varies somewhat with the type of battery and rate of discharge. For example, the end point for stationary batteries is 1.75 volts per cell at the 8-hour rate and for vehicle batteries 1.75 at the 5-hour rate. These voltages have no relation to the open circuit voltage. Only a very small additional capacity can be obtained by discharging the battery further.

The principal factors which affect the capacity of storage cells are: (a) *The amount of material in the cell.* Applying Faraday's law to the equation given above, we find that 3.866 grams of lead, 4.463 grams of lead peroxide and 3.660 grams of sulphuric acid are used per ampere-hour. From two to five times these quantities are actually required, however, since the amount of lead or peroxide which actually takes part in the reactions is only 20 to 40 per cent of that actually present. The maximum output for a complete cell would be about 24 ampere-hours per kilogram, but the best batteries of today give half of this or less.

(b) *Thickness of plates.* Cells with thin plates give somewhat more capacity than those of similar size containing thick plates, particularly

at high rates of discharge. The capacity of the thin plates *per plate* is actually less than that of the thick plates, but it is possible to put more of them in a given space. The thinnest plates now made for aviation batteries are 0.050 inch, while some thick plates for locomotive batteries are more than 3/8 inch.

(c) *Rate of discharge.* A cell which gives 10 amperes for 5 hours can not give 50 amperes for one hour. The expression derived by Peukert applies fairly well to American cells, $I^n t = C$, where I is the current, t the time, and n and C constants that may be evaluated for any type of cell from 2 tests at different values of I. The values for n are usually between 1.2 and 1.5.

(d) *Temperature* has an important part in determining the capacity because it affects the resistance, but more especially because of the marked change in viscosity of the electrolyte and the consequent effect on the rate of diffusion into the pores of the plates. The temperature coefficient of capacity, which must not be confused with the temperature coefficient of voltage, varies with the type of battery and the rate of discharge. A rough approximation is one per cent per degree C decrease with falling temperature.

(e) *Concentration of electrolyte.* The capacity increases with increasing concentration except as certain factors including local action and resistance tend to offset it. The effect of increasing the concentration is particularly noticeable for high-rate discharges. It is obvious that for low rate discharges it is preferable to use a moderate concentration and avoid the effect of local action.

(f) *Porosity of the active material.* The importance of porosity in facilitating the access of electrolyte to the active material is very great. The porosity of the finished plate varies with the material from which it was made and the state of charge. The aggregate porosity of a good plate is a matter of 50 per cent, but the individual pores are little larger than molecular in size.

(g) *Effect of previous discharges.* A hysteresis effect is often observed when measuring capacity following discharges at lower or higher rates. When a higher rate follows a lower rate the capacity will be higher and when a lower rate follows a higher rate the capacity will be lower than normal. This effect is temporary.

(6) *Characteristics of charge and discharge.* When a battery is on charge the current flowing is given by the following formula in which I is the current at any instant, E, the applied electromotive force, and E_c the counter electromotive force of the battery, and R the resistance

of the battery corresponding to the particular value of I. $I = \frac{E - E_c}{R}$

The reason for the high value of E_c during charge is to be attributed to the high concentration of electrolyte in the pores of the plate and the concentration of lead and sulphate ions rather than to resistance which actually decreases as the charge progresses.

Charging may be done by the familiar constant-current method or by the constant-potential method (usually modified by the addition of a small fixed resistance in series with the battery). By the latter method the charging potential is maintained at a fixed value of about 2.3 volts per cell at the battery terminals. The current, initially large, tapers off as the charge progresses. A battery is said to float upon a charging line when the applied voltage is equal to or only slightly greater than the open circuit voltage of the battery and of opposing polarity.

When a battery discharges, the drop in voltage is gradual during the greater part of the time but becomes more rapid (knee of the curve) as the end-voltage is reached. Beyond this point the voltage falls so rapidly that little additional capacity can be obtained. The average voltage during discharge of a vehicle cell is about as follows:

Normal rate (5 hours)	1.95 volts
twice normal rate	1.90
five times normal rate	1.75
ten times normal rate	1.50

The temperature rises more rapidly during charge than during discharge because the reversible and irreversible heat effects are then in the same direction, but the temperature of the room has considerable influence.

Gassing is determined by voltage conditions at the plates during charge and the amount of gas is fixed by the quantity of electricity supplied to the battery in excess of that used for the charging of the plates. At the end of the charge the gas is approximately two parts hydrogen and one part oxygen. An open flame should never be brought into a battery room or near the vent plug of a cell unless it has been "blown out." Gas escaping while the battery is on open circuit is residual gas or the product of local action at the negative plates.

(7) *Resistance.* The internal resistance of a storage battery is small and for many purposes may be neglected. Present day usage, however, often requires a battery to deliver very large currents. The idea that a battery must not be discharged at high rates has now been abandoned. Large currents are required when a tractor begins to pull a heavy load,

or when an automobile is cranked. The central station batteries in one of our large cities have a combined emergency six-minute rating of over 2,000,000 amperes at 125 volts. In such cases the resistance of the battery is important. The external circuit receives maximum power when its resistance is equal to the internal resistance of the battery. The condition for maximum mechanical power is that the back electromotive force of the motor equals one half the open circuit voltage of the battery.

The resistance of a discharged battery may be as much as five times its resistance charged. The values obtained for the resistance vary somewhat with the method used. For engineering purposes the drop-in-potential method using an ammeter and a voltmeter is recommended, although smaller values are obtained by using an oscillograph. Various direct current methods using condensers and the alternating current methods are of some theoretical interest, but no practical importance, and they do not agree in general with the voltmeter-ammeter measurements.

(8) *Efficiency.* The ampere-hour efficiency depends very largely on the completeness of the recharge. A portion of the charging current is wasted as gas and this reduces the efficiency. The normal operating efficiency of a lead battery is 85 per cent but by care in charging and correcting for temperature this may be increased to practically 100 per cent. The voltage efficiency is the ratio of the average voltage of discharge to the average voltage of charge. Under ordinary conditions this is about 85 per cent also. The product of the ampere-hour efficiency and the voltage efficiency gives the watt-hour or energy efficiency. This is usually 70 to 75 per cent, but Bailey (Electrical World *47*, p. 829, 1906) has shown that by eliminating the concentration polarization by making the cycles of charge and discharge as short as 1/60 of a second that the energy efficiency is as high as 98 per cent. The remaining 2 per cent was attributed to ohmic resistance losses. This shows how perfectly reversible the reactions in the lead cells are.

(9) *Sources of trouble.* (a) *Overcharging*, if severe, produces excessive gassing, loosens the active material, particularly of the positive plate, and may give rise to excessive temperatures that are destructive to separators and negative plates. Excessive brown sediment is evidence of frequent overcharging.

(b) *Undercharging* is one of the most common causes of buckling. The lead sulphate occupies more space than the original material and

an excessive amount strains the plate. A fine white sediment is evidence of habitual undercharging.

(c) *Corroded terminals* often prevent the battery from delivering a heavy current, although it may be possible to charge the battery and to draw moderate currents from it. The terminals should be cleaned and wiped with a cloth moistened with ammonia water, and then covered with vaseline.

(d) *Short circuits* may be caused by breakdown of one or more separators, the accumulation of sediment in the bottom of the jars or the growth of a tree-like structure from negative to positive plates. Evidences of short circuit are low open circuit voltage and rapid loss of charge. The only remedy is to open and repair the cell.

(e) *Worn-out plates* are detected by low capacity following charging in which the specific gravity of the electrolyte rises quickly to the proper value. There is no remedy except to replace the plates.

(f) *Electrolyte below the tops of the plates* gives rise to abnormal sulphation. It is generally sufficient to fill the cells with water and allow the battery to continue normal operation.

(g) *Freezing* probably does not damage a charged battery but it seldom occurs. A completely discharged battery may freeze at 20°F or below. The ice crystals plus the sulphate crystals cause some expansion of the active material. Unless the jars are cracked or the plates buckled no great harm is done.

(h) *Impurities in the solution.* See section on electrolyte.

(i) *Defective or corroded plates* may result from defects in manufacture or impurities in the solution. When the active material cracks away from the grid, sulphate intervenes and the plate can not be charged. It is worthless.

(j) *Excessive sulphation* is generally the result of neglect. The sulphate which forms during discharge is easily reduced by charging but may harden if allowed to stand for a considerable time. One remedy is to pour out the electrolyte and fill the cell with pure water, and then put the battery on charge for a prolonged time, and finally adjust the specific gravity to the proper value when charged.

(k) *Reversal* may be caused by over discharge of a cell deficient in capacity as compared with others in the same battery. This results in excessive local action but if the plate is not disintegrated it can usually be restored by charging in the right direction.

(l) *Shrinkage of negatives* is counteracted by the use of expanders incorporated in the active material. Planté negatives which have lost capacity may often be restored by reversal followed by re-reversal.

(m) *Unsteady voltage* is generally caused by loose or corroded contacts, but may be caused by the liberation of residual gas shortly after charging. This effect is transient. It is sometimes desirable to discharge the battery for a short time previous to its use in experiments requiring very steady voltage.

II ALKALINE STORAGE CELLS.

The alkaline storage cell is most familiarly known in this country as the Edison cell, but other forms have been made in European countries. The general character of the reactions is the same in all, but the construction of the parts is quite different. The following description applies to the Edison cells.

The active material of the positive plates is contained in reinforced perforated steel tubes made from nickel-plated sheet steel. Alternate layers of nickelous hydroxide and flake nickel are tamped into these tubes; there are 630 layers in each tube. The tubes are 4-1/2 inches long and 1/4 inch in diameter ("A" plate) or 3/16 inch diameter ("G" plate). The active material is then formed; i.e., oxidized to a higher oxide. The active material of the negative plates is contained in nickel plated steel boxes with perforated sides. The material is put in as the black oxide of iron, with the addition of a small amount of yellow oxide of mercury. The active material is reduced electrolytically to metallic iron. The cell is therefore sometimes referred to as the nickel-iron cell, the nickel oxide being in the positive plates and the iron in the negative plates. Unlike the lead cells the Edison cell is contained in a welded container of nickel-plated steel. Some care is therefore required to avoid short circuits between cells. The cells are mounted in open trays, a space between adjacent cells being provided for insulation. As in the lead cells, there is always one more negative plate than positive plates, but the cell designations refer to the positives; e.g., A6 means that the cell contains 6 positive plates of the A type.

The electrolyte is a solution of potassium hydroxide in water to which a small amount of lithium hydroxide is added. The latter has a beneficial effect on the operation, but is not necessary for the fundamental reactions. At times sodium hydroxide has been used as a substitute, but the potassium electrolyte is preferable. The initial filling of the cells is done with "first fill electrolyte," specific gravity 1.228. Spillage

should be replaced with "refill" electrolyte, sp. gr. 1.210. The electrolyte does not vary in concentration during charge and discharge as in the case of the lead battery, but there is a gradual decline over a long period of time so that it should be renewed when it has fallen to a specific gravity of 1.160. "Renewal" electrolyte has a sp. gr. 1.248. These electrolytes vary somewhat as to the lithia content as well as specific gravity. The resistivity of these solutions is somewhat higher than the sulphuric acid solutions used in lead batteries. Distilled water should be used for flushing. The vent plugs should be kept closed to avoid the absorption of carbon dioxide from the air. Gases generated within the cells can escape through the vent plugs when necessary.

The reactions of the Edison battery are not as definitely known as those for the lead batteries. Probably the final oxidization product of the positive plate is nickel peroxide, but this is not stable as Foerster (Elektrochemie Wasseriger Lösungen 3d ed. p. 270, 1922) has shown. Probably the NiO_2 decomposes to Ni_2O_3 with the evolution of oxygen and the loss of about 0.13 volt. The equation for the reaction in its simplest form is $Ni_2O_4+Fe \rightleftarrows 2\ NiO+FeO$ although it is probable that both the nickel and iron are hydrated. During discharge the water consumed at the positive plate is balanced by the water liberated at the negative and the two molecules of potassium hydrate formed at the positive just balance the two that disappear, hence the electrolyte as a whole does not appear to change and no symbol for it appears in the reaction. The net result is a transfer of oxygen from one plate to the other.

Edison batteries of the more familiar types are normally rated for the 5-hour discharge. The proper end point voltages (discharge) are

normal rate	0.90 volt per cell
twice normal rate	.80 " " "
thrice normal rate	.70 " " "

The amount of nickelous hydroxide used in filling the tubes is 4.64 times the theoretical requirement which is 1.729 grams per ampere-hour. The amount of iron required per ampere-hour is 1.042 grams but the amount in the pockets averages 5.78 times this amount.

The relation of the capacity to the rate of the discharge of Edison batteries is quite different from that of lead batteries. The full capacity of an Edison battery may be obtained irrespective of the rate of discharge, but at a lower voltage for the higher rates. This is because the active material of the positive plate which limits the capacity contracts during discharge. There is nothing analogous in the operation

of the Edison battery to the formation of sulphate which clogs the pores of the lead batteries.

The capacity of Edison batteries is decreased as the temperature is decreased, but the change is not gradual. The Edison batteries have a critical temperature which varies with the rate of discharge. Below this critical temperature (electrolyte temperature) the output is small. At the normal rate of discharge the critical temperature is about 5°C. There is sufficient ohmic resistance so that the heat liberated within the cell is ordinarily sufficient to keep it in operation. Sluggishness may occur when the battery is allowed to stand idle for an extended time. This may be overcome by discharging it to zero voltage, short circuiting it for five hours and then charging for 15 hours, discharging again without short circuiting and charging for 12 hours.

While Edison batteries may be charged by the constant potential method, it is better to use the constant current method, maintaining the starting rate to the end as they do not operate well on low rate charging.

The internal resistance of Edison storage cells is slightly higher than for lead cells of corresponding sizes. The resistance begins to increase rapidly as the end of the discharge is reached. The ampere-hour efficiency is about 82 per cent and the watt-hour efficiency 56 to 60 per cent.

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BUREAU OF STANDARDS,
WASHINGTON, D. C.

THE RESIDUAL ABERRATIONS IN COMA-FREE CEMENTED OBJECTIVES OF THE CROWN-LEADING TYPE

G. W. Moffitt and O. K. Kaspereit

A simplified and direct method for use in the designing of coma-free cemented telescope objectives of the crown-leading type has been given in a recent paper.[1] The inverse features of the well-known method described in a generally excellent paper by Harting[2] were almost entirely eliminated, so that the new method is more straight-forward as far as the designer is concerned. Certain simple and useful empirical relations were found to exist among the constants of the glasses that will combine to form a lens of this kind. The use of these relations—either through substitution in linear formulas, or by the employment of graphical methods—makes it possible to select pairs of glasses that will serve, to determine the value of the intermediate radius of curvature, and to make a fair estimate of the magnitude of the residual aberrations for those cases in which the glasses are not quite the most suitable.

A survey of available glasses reveals the existence of many possible combinations that will give a cemented doublet objective of the coma-free type. Therefore it is logical to consider the question of the variations in the residual aberrations among the members of such a series of objectives. A knowledge of the variation in the zonal aberration is especially important since it is this zonal aberration that may limit the relative aperture. Harting gives some attention to the problem for a number of random examples in the paper already mentioned—a matter to which we will return later. In order to obtain systematic data on this question of the variations in the residual aberrations with change of glasses, we have made an investigation of the subject.

Because of the distribution of existing glasses on the glass chart, and of the limitations of practical designing, it proved to be advantageous for the purposes of this study to reverse the procedure followed in the former paper on the designing of the coma-free objective, and to ask the question, "What crown, or crowns, will combine with a given ordinary flint to form a corrected objective of the cemented coma-free type?" Usually only those ordinary flints of mean index between

[1] G. W. Moffitt, J.O.S.A. & R.S.I., *11*, p. 147, 1925.
[2] H. Harting, Zeitschr. f. Instrkde, *18*, pp. 357-380, 1898.

1.60 and 1.65 would be selected for practical purposes. Therefore it seemed particularly important to study the characteristics of objectives

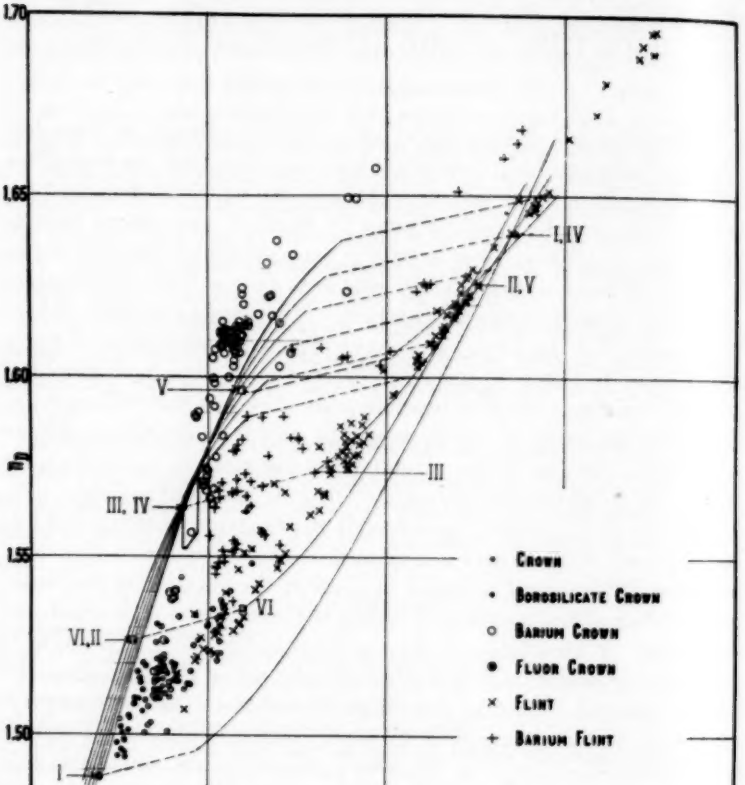

Fig. 1. *The glass chart, to which has been added the series of lines used in locating the region of crowns suitable for combining with the ordinary flints.*

made from these flints paired with crowns of widely different types. Using the method of the former paper a line was drawn on the glass

chart representing the crown glasses that would combine with an ordinary flint of mean index 1.60. Similar curves were drawn for ordinary flints of mean index 1.61, 1.62, and so on to 1.65. See Fig. 1. A crown glass falling within the area covered by this family of curves will combine with at least one ordinary flint between the limits stated. This does not mean that a crown *must* fall within this region in order to be used at all in a coma-free objective, for there are many other flints that might be used,—some of the barium flints, for example. But it does mean that they are the crowns and flints most likely to be used.

Four crowns rather uniformly distributed along the middle line of the crown area were selected, the assumption being that objectives using such a series of crowns would probably run the gamut of variations in characteristics. Wherever possible actual glasses were taken. The curve of suitable flints for each of these crowns was then drawn on the chart. A study of these curves lead to the selection of the flints. It will be noticed that the flint curve for crown I (a real fluor crown) lies mostly in the region to the right of the straight line along which the ordinary crowns and flints fall, but that it cuts through into the region of real glasses at about 1.635 where there are several ordinary flints. The one designated by I was selected to pair with the fluor-crown. The non-existent glass, II, in a region unoccupied by actual glasses, would combine with flints of mean index 1.61 to 1.63 and also with some very light flints around 1.535. Flint II was selected. Later, however, VI was added in order to secure additional data on a point to be discussed later. Glass III occupies a strategic position on the chart. Since the crown curves are all very close together in that region this crown will combine with a large number of ordinary flints between the limits 1.57 and 1.65 or higher. Flints IV and III were selected, the one to form a member of the main series, the other to determine the effects of a change in flint when the crown remains unchanged. Crown V combines with the ordinary flints between 1.62 and 1.64, and it might be matched with a number of barium flints. Flint V was selected. These pairs constitute the main series selected for the purposes of the investigation.

The spherical aberration curve was determined for each of the fully corrected objectives. Equivalent focal length was held at 100, crown thickness at 6.0, flint thickness at 2.0, and the spherical aberration was adjusted to zero at $h = 7.0$. All members of the series were practically coma-free. The dimensions and some of the properties of this series of objectives are arranged in Table 1, while Fig. 2 shows the spherical aberration curves.

There is a surprising similarity in the curves for objectives I, II, IV and V. In fact, there is little to choose between them, widely different as they are in glasses and in form. Objective III, however, shows a marked increase in zonal aberration. It is made of the same crown as that in objective IV but the flint is of a lower mean index. This leads to a smaller *difference* of mean index for the pair of glasses. In order to make doubly certain of this point, objective VI composed of glasses having a still smaller index difference was designed and its spherical aberration curve determined. The curve is the crookedest of the lot.

This is not in accordance with the conclusion reached by Harting in the paper already mentioned, in which the external curvature is considered to be the determining factor. He makes the assumption that

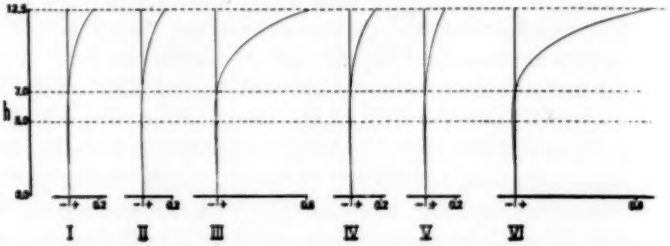

Fig. 2. *Spherical aberration curves for the corrected objectives from the representative glass pairs selected for the investigation.*

a small external curvature leads to small zonal errors. That an error of this kind should have crept into a paper so carefully done and so excellent in other respects as that by Harting, is unfortunate. It is a good illustration of the type of error that geometrical opticians are constantly in danger of making unless they resort to experimental verification (actual calculation of practical systems). In Table 1, it will be seen that the external curvature does not control the zonal aberration, some of the best lenses having a comparatively high external curvature. On the other hand it will be noticed that the crookedness of the spherical aberration curve increases as the difference in the mean indices of the glasses decreases. This is clearly shown in Fig. 3, in which all six of the objectives are represented. It is remarkable that six objectives so utterly unrelated to each other should give a fairly smooth curve when plotted in this way. Obviously the index difference is the controlling factor in determining the crookedness of the spherical aberration curve, although undoubtedly other factors may play a minor part.

TABLE 1.

Objective	$1^{E}D$	$2^{E}D$	n Diff.	Δn_1	Δn_2	r_1	r_2	r_3	$1/r_1-1/r_3$	F_F-F_D
I	1.4881	1.6398	.1517	.00692	.01849	57.93	−41.84	−120.85	.02554	.0574
II	1.5265	1.6250	.0985	.00785	.01750	62.02	−39.33	−152.28	.02269	.0462
III	1.5638	1.5738	.0100	.00928	.01299	61.00	−19.84	−437.24	.01868	.0774
IV	1.5638	1.6406	.0768	.00928	.01856	62.39	−38.13	−210.77	.02077	.0690
V	1.5963	1.6250	.0287	.01094	.01750	63.74	−28.90	−372.01	.01838	.0560
VI	1.5265	1.5354	.0087	.00785	.01099	57.15	−18.41	−407.01	.01996	.0527

Consideration of Fig. 3 shows that so long as the index difference for the glass pair is greater than about 0.04, there is little to choose among the possible coma-free objectives but that for smaller values the zonal aberration increases rapidly. Therefore an index difference smaller than 0.04 might well be avoided in objectives that must work at large relative apertures.

The secondary chromatic aberration of the series of objectives turned out to be less variable than the elementary algebraic theory would lend one to suspect. In Table 1 the difference in position of the common focus for C and for F and that for D is quite constant, although the elementary algebra would lead one to expect greater residuals when

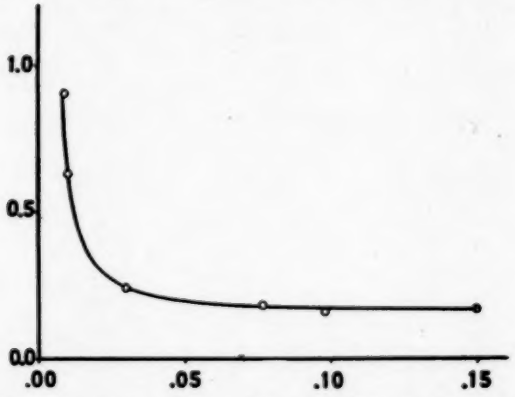

FIG. 3. *Variation of zonal error with index difference. Abscissae,—$(n_F - n_C)$. Ordinates,—Over-connection of spherical aberration at $h=12.5$ when zero at $h=7.0$.*

the index difference is large. Moreover, the values of the residual color aberration are quite small. As far as this residual aberration has to do with the design of small objectives of this kind, there is nothing to choose among the possible glass pairs leading to satisfactory correction of the other aberrations. Certainly the residual chromatic aberration would seldom, if ever, enter as the determining factor in the selection of a glass pair.

The results thus far given are for objectives intended to serve without any prism glass in the image space. But since the majority of small objectives are used in instruments containing prism glass it becomes desirable to ascertain what changes, if any, the introduction of prism glass would make in the conclusions already reached. In order to determine the effect of the introduction of prism glass on the pairing of

glasses for the coma-free type of objective, objective IV was redesigned to serve with 100 mm of prism glass in the image space. This necessitated a lowering of index of about 0.018 in the crown glass, holding the flint glass and Δn for the crown as before. If it may be assumed that a similar lowering of the crown index would prevail throughout that part of the chart covered by the crown glass curves in Fig. 1, we may say that the introduction of prism glass lowers the area of suitable crowns, the amount of this lowering depending on the thickness of the prism glass in the system, and being about 0.018 when the prism glass has an excessive thickness equal to the focal length of the objective. The effect of such a lowering is to bring many borosilicate and light barium crowns within the area of crowns suitable for the ordinary flints already selected for the purpose of this discussion.

It will be noticed that the area of suitable crowns is very constricted in one place. Any glass falling within this constriction is especially suitable for use in the coma-free type of objective, providing, of course, that its other qualities are satisfactory. As prism glass is added to a system this constriction moves downward on the chart sweeping out a small area having the form of a parallelogram. Unfortunately there are very few glasses within it, although it is near the light barium crown group. It would seem to be worth while to obtain more glasses within the area indicated by the parallelogram.

If possible, the displacing of some borosilicate crowns slightly to the left or upward on the glass chart would also be desirable.

In the region of the fluor-crowns, too, it might sometimes prove advantageous to have more glasses, such as *I*, since these glasses pair well with a good group of ordinary flints to give combinations having a high Δn ratio. While such a combination does not seem to have any advantage over the parallelogram group—except that it leads to an objective having flatter curves, no doubt a desirable feature in certain eyepieces—yet the very fact that it would make available another possible solution of the problem might be, of itself, important.

FRANKFORD ARSENAL, PA.
MARCH, 1925.

Magnetic Moments of the Atoms of Various Metals.—Repitition and continuation of the very important experiments on this quantity by Gerlach and his associates (*cf.* This Journal *10*, p. 282) which give absolute values for the magnetic moments of individual atoms and apparently demonstrate also that the atoms instantly assume particular orientations determined by quantum conditions

when they enter a magnetic field. Additional determinations for silver give the value 5690 for its atomic moment; 5600 is the moment of an electron orbit of angular momentum $h/2\pi$. i.e., "one Bohr magneton." The first data for *iron* indicate that its atoms have no magnetic moment (!) while those for *nickel* indicate a moment of two or more Bohr magnetons. The apparatus and method are described in extreme detail; the difficulties of converting invisibly thin deposits of metal on the glass receiving plate into visible ones and interpreting their aspect are stressed. Many photographs are reproduced, some of them quite striking, although it is difficult to see how a greater accuracy than 10 per cent in determining the magnetic moment from them can be attained.—[W. Gerlach (Frankfort); Ann. d. Phys. *76*, pp. 163-197, 1925.]

KARL K. DARROW

Illuminating engineering.—Edited by Francis E. Cady, of the National Lamp Works of the General Electric Company, and Henry B. Dates, of the Case School of Applied Science. xiii+486 pp. Published by John Wiley and Sons, Inc., New York City, New York. $5.00.

This book is based upon the material given, for the past six years, in a course in Illuminating Engineering at the Case School of Applied Science in cooperation with the National Lamp Works of the General Electric Company. The course is given to undergraduates and consists of three lectures and one laboratory period per week throughout the year. The lectures are given by specialists in the several fields, and the book, therefore, is a collaboration of a group of some of the most prominent illuminating engineers in the country.

The subject matter is divided into fourteen chapters: viz., The Physics of Light Production; Light Sources; Photometry; Physiological Optics; Principles of Illumination; Light, Shade, and Color; Daylight; Residence Lighting; Lighting of Public Buildings; Commercial Lighting; Industrial Lighting; Sign and Display Lighting; Street Lighting; Light Projection. Each chapter is well subdivided into sub-titles and captioned paragraphs. There is also a carefully prepared index. Comparatively few references to original sources are given throughout the text, but a valuable feature of the book, for pedagogical purposes as well as for reference, is a list of references for collateral reading at the end of each chapter.

One single example will serve to illustrate the care with which the book has been prepared: In the chapter on Principles of Illumination is included a series of thirty-two samples of paints such as are used for interior decoration, with coefficients of reflection (for Mazda light) given for each.

With such a concise, yet complete, book available there should no longer be excuse for ignorance of this vital subject of Good Lighting.

F. K. RICHTMYER

A NEW FORM OF RADIOMETER

By R. L. Hanson

One of the difficulties which the experimenter encounters in working with a gold leaf electroscope at atmospheric pressures is the control of convection currents set up within the instrument whenever a slight change of temperature occurs in any one part. While working with a sensitive Bumstead electroscope, Becker[1] (and later Read[2]) were troubled by a drift of the gold leaf which took place as soon as he approached the instrument. Even the air currents about the room were such that he found it necessary to place baffle plates in his electroscope to protect the leaf. Farwell[3] observed a similar drift of the leaf of a Wilson tilting electroscope. He placed a thermopile near one side of the electroscope and allowed radiant energy from an electric lamp to strike the two instruments. On varying the current through the lamp and observing the deflections of the gold leaf and galvanometer in the thermopile circuit he found that the deflections of the two were similar, that is, the two deflections plotted against time gave similar curves. Thus he proved to his satisfaction that the drift of the leaf was due to convection currents caused by uneven heating of the case. The phenomenon was so marked that it seemed worth while to study the possibility of constructing an instrument utilizing this effect for the measurement, or at least the detection, of radiant energy.

The apparatus first used for the study of this effect was a cylindrical brass box containing a very narrow gold leaf G, suspended from a sulphur plug S (Fig. 1). Radiant energy was allowed to enter the thin window W, and strike an absorbing target which consisted of silver foil .025 mm thickness coated with lampblack. The radiation absorbed by the target raised its temperature and set up convection currents which moved in the general direction shown in the figure, blowing the lower end of the leaf to one side. The sensitivity was surprisingly high, the heat from the hand held several feet away from the window being sufficient to move the leaf several divisions of the microscope which was used for observing the deflection.

In order to increase the sensitivity electrodes E, E, charged to potentials of about $+200$ and -200 volts respectively were placed on opposite

[1] Becker, Phys. Rev., *20*, p. 134-47; 1923.
[2] Read, Phys. Rev., *21*, p. 368; 1923.
[3] Farwell, Am. Jour. of Sc., *37*, p. 319; 1914.

sides of the leaf as shown. The leaf could be earthed by means of a spring contact *B* and then insulated again. As the leaf swung toward one of the electrodes, the electrostatic force of attraction increased and partially neutralized the force of gravity on the leaf. The sensitivity could in this way be increased, but the field was such that the leaf became unstable before high sensitivity was reached. Other arrangements of the electrodes were tried but those which gave a field of the proper shape seemed to interfere with the free motion of the convection currents.

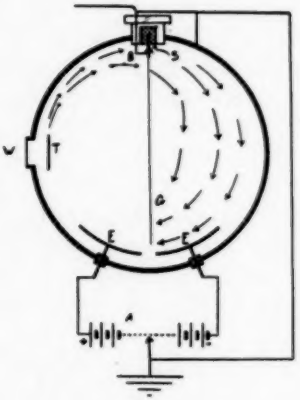

FIG. 1

A much more satisfactory instrument was later devised in which the gold leaf was replaced by a Rayleigh disk[4] suspended by a very fine quartz suspension. The construction of this instrument is shown in Fig. 2. A disk *D*, of aluminum foil about 3x4 cm was suspended as near as possible to the top of the chamber which was about twenty centimeters in diameter and ten centimeters deep. It had previously been found by means of a glass chamber of similar shape, filled with smoke and equipped with an electric heating coil in the position of the target *T*.

[4] Barton, Text on "Sound," p. 365.
Konig, Weid. Ann., *43*, p. 43; 1891.
Thomson and Tait, Theor, Physics.
Kirchoff, Mechanik.
Stewart and Stiles, Phys. Rev. N. S. *1*, p. 311, 1913.
Rayleigh, Phil. Mag., *14*, p. 186; 1882.
Zernov, Ann. der Physik, *26*, p. 79; 1908.
Grimsehl, Weid. Ann., *34*, p. 1028; 1888.
Lebedew, Weid. Ann., *62*, p. 163; 1897.

that the flow of the gas was confined to a rather concentrated stream traveling with a relatively high velocity at the top of the chamber, but spread out in all directions and traveled more slowly as it left the top. For this reason the disk was placed at the top. The absorbing target T, which was just large enough to catch all the radiation entering the window, was made of silver rolled to a thickness of .025 mm and coated with lampblack by holding it over burning camphor. This gave a very satisfactory dull surface. The whole instrument was nickel plated to reflect any radiation which might strike it at any place other than the window. The deflections of the disk were observed by means of a spot

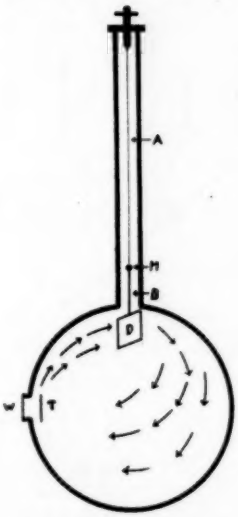

FIG. 2

of light reflected by the mirror M to a scale, M being rigidly attached to the disk D by means of a coarse quartz fiber. Before striking the mirror the light was passed through a .32 N solution of anhydrous cupric chloride in absolute alcohol which transmits radiation between .515μ and .590μ, to filter out the heat radiated from the light source, thus at the same time filtering out the maximum amount of heat energy and a minimum of light. The main body of the instrument was further protected from the radiation from the light source by placing the mirror outside the instrument proper as shown in the drawing. The whole instrument was placed in a thermally insulating jacket to protect it from air currents which are invariably present in any room. Building

jars were eliminated very successfully by placing the instrument in a heavy frame and suspending it by a coil spring.

The general set up was that shown in Fig. 3. A carbon lamp *S* calibrated against a radiation standard obtained from the Bureau of Standards served as a source of radiant energy. A black body *B*, made by lining a box with black velvet was placed behind *S*. Screens *A* served to cut off stray radiation which might be reflected from the walls of the room to the instrument. The energy flux incident upon the receiving surface was varied by means of a sector disk not shown in the diagram. The shutter *F* could be raised and lowered to completely cut

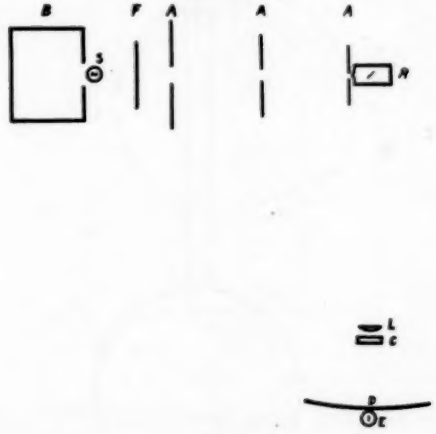

FIG. 3

off the radiation falling on the window of the instrument. The optical system by which the deflections of the Rayleigh disk were read consisted of a single filament galvanometer lamp *E*, a cell *C* containing the filtering solution, and a lens *L*, which threw an image of the filament on a scale *D*, after reflection from the mirror of the instrument.

With the instrument well shielded from the observer, readings were taken for the deflections as the energy striking the absorbing target were varied. If care was taken not to set up undue currents about the room the disk could be brought back to the same zero reading to within one millimeter. Readings were taken both by returning to the zero position each time and by increasing the radiation by steps with the same results. Fig. 4 shows the deflections in centimeters on a scale at two meters, plotted against the watts energy flux striking the absorbing target.

This shows that at least for the range used the relation between these is linear. The instrument thus gives a sensitivity of approximately 8×10^{-9} watts/mm. Hence a standard candle at a distance of one meter from the target would give a deflection of 980 cm on a scale at one meter. This divided by the area of the receiving surface, expressed in square millimeters, becomes 6.5 cm, i.e., the deflection per square millimeter of receiving surface, given by a standard candle and scale at one meter would be 6.5 cm. The highest corresponding sensitivity yet obtained with a Nichols radiometer as given by Coblentz[5] is of 71 centimeters.

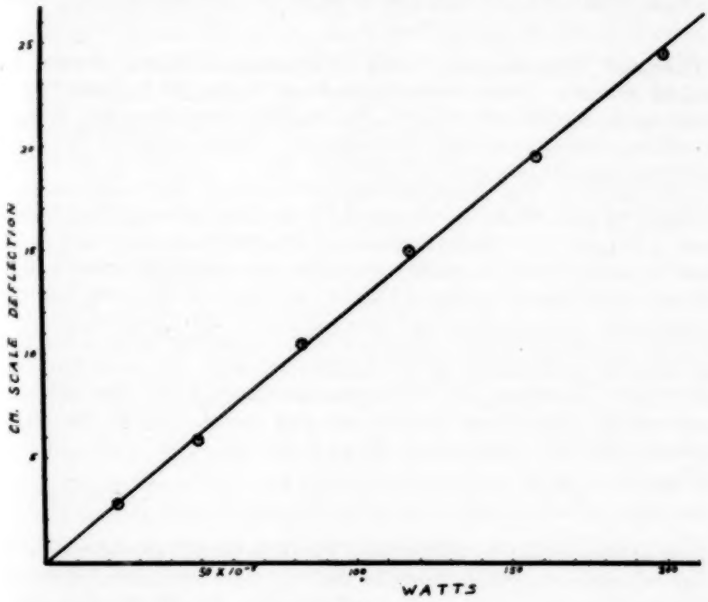

FIG. 4

The results here shown for this instrument by no means represent the maximum sensitivity obtainable. It seems very probable that by further refinements such as a finer suspension for the disk, a lighter disk, etc., the sensitivity could be pushed to ten or twenty times that given above. In this experiment, air at atmospheric pressures was used within the instrument, while it seems probable that some other gas at a different pressure would increase the sensitivity manyfold.

[5] Coblentz, Bureau of Standards Bull., *4*, p. 391; 1908.
Coblentz, Jour. Op. Soc. Am., *7*, p. 445; 1923.

This instrument like the Nichols radiometer has the disadvantage of a very long period. It took the disk about ten minutes to come to a steady state and about the same time to return to the zero point again. The surprising thing about the whole experiment was that such a high sensitivity could be obtained with such a simple apparatus.

For the success of this work the author is indebted to Prof. F. K. Richtmyer who suggested the problem and under whose supervision the experiment was carried on.

CORNELL UNIVERSITY,
ITHACA, N. Y.

Thermal Conductivity and Wiedemann-Franz Ratio for heated Metals.—Measurements made on Al, Cu, Mg, Zn and Ni at temperature ranging up to 700°C by establishing a heat-flow down a bar of the metal from an electrically heated wire embedded at its center, and measuring the temperature-gradient. The description of the experiments illustrates the frequently made assertion that measurements of thermal conductivity are among the most difficult in physics. Sometimes a datum refers to a condition in which the temperature of the metal varies by as much as 100° across the segment under examination. The values of thermal conductivity decline slowly with increasing temperature, except for Al for which they rise slowly; the values of electrical resistance increase linearly, except for Ni for which the curve is crooked. The value of the Wiedemann Franz ratio (thermal over electrical conductivity) is proportional to temperature for Cu, Mg, Zn and rises more rapidly than T for aluminium, and nickel.—[F. H. Schofield, National Physical Laboratory; Proc. Roy. Soc. *A107*, pp. 206-227, 1925].

KARL K. DARROW

Law of Motion of an electrified Particle in a Gas.—Experiment to see whether electrified particles such as used in determining e by Millikan's method (in this case, particles of Se, Pt, Ag and Cu) would respond to an alternating field of frequency 50 in the manner to be expected from their response to a steady field. They did. The author also studies the photoelectric emission from selenium particles when irradiated, measuring the time-interval between beginning of irradiation and emergency of the first photoelectron, and the total positive charge acquired by the particle up to the time when its positive potential becomes so great that no more photoelectrons can escape. Such studies have already been performed by Meyer and Gerlach for steady applied fields, and the observations with alternating fields do not seem to add anything essentially new.—[Yü Chen Yang. Zürich: Ann. d. Phys. *76*, pp. 333-368, 1925].

KARL K. DARROW

THE DESIGN OF A CAM FOR AN X-RAY SPECTROGRAPH

JAMES B. FRIAUF

In photographing x-ray spectra by the rotating crystal method it is frequently desirable that the crystal be given such an oscillatory motion that it rotates through a given angle with constant angular velocity, then reverses its direction of rotation and returns with the same constant speed. The same type of motion may be useful in other cases and can be obtained from a suitably designed cam rotating with constant angular velocity. This article contains an account of an analytical method which enables the computation of the proper profile for the cam. This could also be done by graphical construction which was the method used for the spectrograph designed by Dickinson and described by Wyckoff.[1] In these spectrographs the distances designated by R_1 and R_2 in this article have been made equal.

In Fig. 1, O represents the axis, perpendicular to the plane of the paper, about which the cam rotates with constant angular velocity in the direction indicated by the arrow. A represents the axis of rotation of the crystal holder, likewise perpendicular to the plane of the paper, and ACB the curved lever which rests upon the cam at B and serves to rotate the crystal holder. R_1 is the distance AO and R_2 the distance AB. r_1 and r_2 are the least and greatest radii respectively of the cam, and r is the radius of the cam at an angle α with the least radius r_1. The angle from OA to r_1 is ωt where ω is the angular velocity of the cam and t the time measured from the coincidence of r_1 with OA. The meaning of the angles θ and φ is clear from the figure.

It is evident from the figure that

$$r^2 = R_1^2 + R_2^2 - 2R_1R_2\cos\theta \tag{1}$$

from which

$$\cos\theta = \frac{R_1^2 + R_2^2 - r^2}{2R_1R_2} \tag{2}$$

The smallest angle θ_1 occurs when the cosine is greatest, that is, when r is least, and the largest angle θ_2 when the cosine is the smallest, that is, when r is largest (as long as θ is less than 180°).

[1] R. W. G. Wyckoff, The Structure of Crystals, p. 163

$$\cos\theta_1=\frac{R_1^2+R_2^2-r_1^2}{2R_1R_2} \tag{3}$$

$$\cos\theta_2=\frac{R_1^2+R_2^2-r_2^2}{2R_1R_2} \tag{4}$$

The angle β through which the crystal holder rotates is then given by the relation

$$\beta=\theta_2-\theta_1 \tag{5}$$

When convenient values of R_1, R_2 and r_1 have been chosen, θ_1 can be determined from equation (3) and r_2 can then be calculated from equation (4) by putting $\theta_2=\theta_1+\beta$ where β is the angle through which it is desired to rotate the crystal holder.

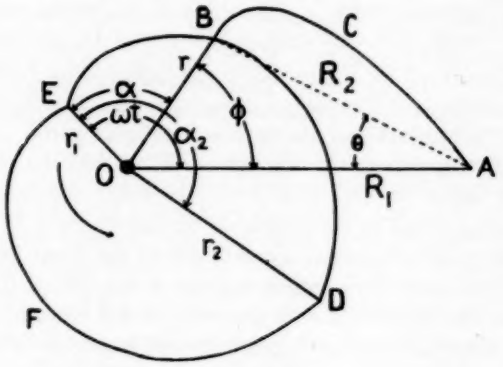

FIG 1.

Differentiating equation (2) gives

$$-\sin\theta\frac{d\theta}{dt}=-\frac{2r\dfrac{dr}{dt}}{2R_1R_2} \tag{6}$$

which can be written

$$\sin\theta\frac{d\theta}{dt}=\frac{r}{R_1R_2}\frac{dr}{d\alpha}\frac{d\alpha}{dt} \tag{7}$$

But it is clear from the figure that $\alpha=\omega t-\varphi$ which gives when differentiated

$$\frac{d\alpha}{dt}=\omega-\frac{d\varphi}{dt} \tag{8}$$

so that equation (7) becomes

$$\sin\theta\frac{d\theta}{dt}=\frac{r}{R_1R_2}\frac{dr}{d\alpha}\left(\omega-\frac{d\varphi}{dt}\right) \tag{9}$$

The quantity $\frac{d\varphi}{dt}$ can be expressed in terms of $\frac{d\theta}{dt}$ in the following manner. It is evident that

$$R_2^2=R_1^2+r^2-2rR_1\cos\varphi$$

giving

$$\cos\varphi=\frac{R_1^2-R_2^2+r^2}{2rR_1} \tag{10}$$

Differentiating this gives

$$\sin\varphi\frac{d\varphi}{dt}=\left(\frac{R_1^2-R_2^2}{2r^2R_1}-\frac{1}{2R_1}\right)\frac{dr}{dt} \tag{11}$$

But

$$\sin\varphi=\frac{R_2}{r}\sin\theta$$

while equation (6) gives

$$\frac{dr}{dt}=\frac{R_1R_2}{r}\sin\theta\frac{d\theta}{dt}$$

Substitution of these values in equation (11) then gives

$$\frac{d\varphi}{dt}=\left(\frac{R_1^2-R_2^2}{2r^2}-\frac{1}{2}\right)\frac{d\theta}{dt} \tag{12}$$

When this value of $\frac{d\varphi}{dt}$ is put into equation (9) the result is

$$\sin\theta\frac{d\theta}{dt}=\frac{r}{R_1R_2}\frac{dr}{d\alpha}\left[\omega-\left(\frac{R_1^2-R_2^2}{2r^2}-\frac{1}{2}\right)\frac{d\theta}{dt}\right]. \tag{13}$$

It is desired to have the angular velocity of the crystal holder constant and of the same magnitude in both directions. Consequently, the crystal holder must rotate through the angle β (measured in degrees) while the cam rotates through 180°. Since the two angular velocities

are constant they must be in the same ratio as the angles through which the crystal holder and the cam rotate in the same time. Hence,

$$\frac{\frac{d\theta}{dt}}{\omega} = \frac{\beta}{180}$$

and

$$\frac{d\theta}{dt} = \frac{\beta\omega}{180} \tag{14}$$

Substituting this value for $d\theta$ in equation (13) and making a few simplifications we have

$$\sin\theta = \frac{r}{R_1R_2}\frac{dr}{da}\left(\frac{180}{\beta}+\frac{1}{2}-\frac{R_1^2-R_2^2}{2r^2}\right) \tag{15}$$

When the value of $\sin\theta$ in terms of r is introduced by using $\sin\theta = \sqrt{1-\cos^2\theta}$ there results

$$R_1R_2\,da = \frac{r\,dr\left(\frac{180}{\beta}+\frac{1}{2}-\frac{R_1^2-R_2^2}{2r^2}\right)}{\sqrt{1-\left(\frac{R_1^2+R_2^2-r^2}{2R_1R_2}\right)^2}} \tag{16}$$

as the differential equation giving the relation between r and a for the part *EBD* of the cam. The introduction of the substitution

$$\mu = \frac{R_1^2+R_2^2-r^2}{2R_1R_2}$$

reduces the integrals to forms which can be found in the tables. The result of the integration is

$$a+C = \left(\frac{180}{\beta}+\frac{1}{2}\right)\cos^{-1}\frac{R_1^2+R_2^2-r^2}{2R_1R_2} + \frac{1}{2}\sin^{-1}\frac{(R_1^2-R_2^2)^2-(R_1^2+R_2^2)r^2}{2R_1R_2r^2} \tag{17}$$

The constant of integration C can be determined from the condition that $r=r_1$, when $a=o$ and is found to be

$$C = \left(\frac{180}{\beta}+\frac{1}{2}\right)\theta_1 + \frac{1}{2}\sin^{-1}\frac{(R_1^2-R_2^2)^2-(R_1^2+R_2^2)r_1^2}{2R_1R_2r_1^2} \tag{18}$$

The constant of integration having been determined, equation (17) can be used to calculate values of the angle α corresponding to different values of r and the profile of the cam for the part *EBD* can be drawn. The angle α_2 corresponding to the greatest radius r_2 is found to be

$$\alpha_2 = 180 + \frac{\beta}{2} + \frac{1}{2}\sin^{-1}\frac{(R_1^2-R_2^2)^2-(R_1^2+R_2^2)r_2^2}{2R_1R_2r_2^2} - \frac{1}{2}\sin^{-1}\frac{(R_1^2-R_2^2)^2-(R_1^2+R_2^2)r_1^2}{2R_1R_2r_1^2} \tag{19}$$

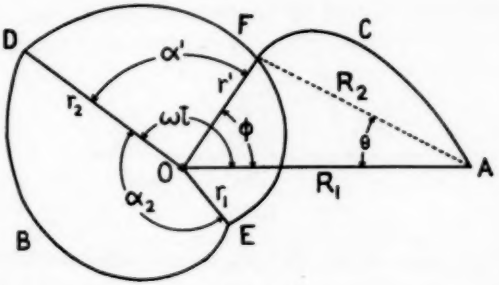

FIG. 2

The profile of the remainder of the cam can be determined in a sim lar way. In Fig. 2 r' represents the radius of the cam at an angle α' with the greatest radius r_2. The only difference with the preceding case is that $\frac{d\theta}{dt}$ is now negative

$$\frac{d\theta}{dt} = -\frac{\beta\omega}{180}$$

When this is substituted in equation (13) the resulting differential equation is

$$R_1R_2d\alpha' = \frac{r'dr'\left(\frac{180}{\beta}-\frac{1}{2}+\frac{R_1^2-R_2^2}{2r'^2}\right)}{\sqrt{1-\left(\frac{R_1^2+R_2^2-r'^2}{2R_1R_2}\right)^2}} \tag{20}$$

Integration gives

$$C'-\alpha' = \left(\frac{180}{\beta}-\frac{1}{2}\right)\cos^{-1}\frac{R_1^2+R_2^2-r'^2}{2R_1R_2} - \frac{1}{2}\sin^{-1}\frac{(R_1^2-R_2^2)^2-(R_1^2+R_2^2)r'^2}{2R_1R_2r'^2} \tag{21}$$

where the constant of integration C' determined from the condition that $r'=r_2$ when $a'=o$ is found to be

$$C'=\left(\frac{180}{\beta}-\frac{1}{2}\right)\theta_2-\frac{1}{2}\sin^{-1}\frac{(R_1^2-R_2^2)^2-(R_1^2+R_2^2)r_2^2}{2R_1R_2r_2^2} \tag{22}$$

The values of a' for different values of r' can now be calculated and the complete profile of the cam drawn. It is evident that a' must be equal to $360-a_2$ when $r'=r_1$ and this is found to be the case by equation (21).

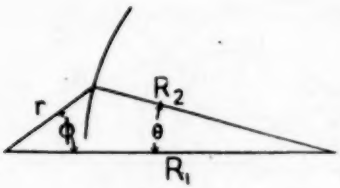

FIG. 3

The equations show clearly that the cam is unsymmetrical and that in general the angle between the least and greatest radii is not equal to 180°. The reason for this is due to the fact that as θ changes the angle φ also changes and the line OB accordingly moves forward or backward. It is desired that the change from the least angle θ_1 to the greatest take place while the cam rotates through 180°. Consequently,

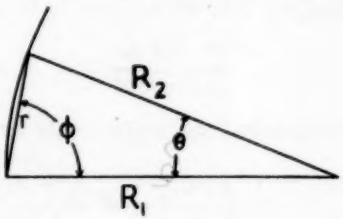

FIG. 4

the point B of the lever must make contact with the greatest radius r_2 when the cam has turned 180° from the initial position in which B makes contact with the least radius r_1. If φ has decreased by the amount φ_0 in this half revolution of the cam, the angle a_2 from the least to the greatest radius must be $180°+\varphi_0$ measured in the direction EBD, while if φ has increased by the amount φ_0' the angle a_2 must be $180°-\varphi_0'$. As shown in Figs. 3, 4 and 5 the locus of B is on a circle

having a radius R_1 and a center at A. When $R_2 > R_1$ as in Fig. 5, or when $R_2 = R_1$ as in Fig. 4 it is evident from the figures that φ decreases as θ increases. The angle α_2 is therefore greater than 180° for these cases. But when $R_2 < R_1$ as in Fig. 3 it is clear that starting from the initial position shown in the figure, φ at first increases to a maximum and then decreases as θ increases. In this case α_2 may be either greater or less than 180° depending upon the values chosen for R_1, R_2, r_1 and β, the angle through which the crystal holder is to be rotated. The cam drawn in Figs. 1 and 2 is calculated to rotate the crystal holder through 30°. R_1, R_2 and r_1 were chosen to be 12, 10 and 3 cm respectively. In this case α_2 is less than 180° being equal to 166.9°. $r_2 = 8.05$ cm.

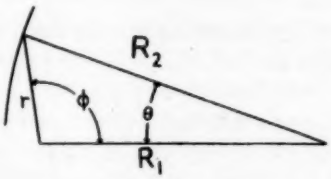

FIG. 5

The equations given make it possible to determine the proper profile to be given to the cam for general values of R_1 and R_2. When the apparatus can be so constructed that R_1 and R_2 can be made equal, the equations become much simpler and the computation of the profile is much facilitated. When $R_1 = R_2 = R$ equation (17) reduces to

$$\alpha + C = \left(\frac{180}{\beta} + \frac{1}{2}\right) \cos^{-1} \frac{2R^2 - r^2}{2R^2} + \frac{1}{2} \sin^{-1}\left(-\frac{2R^2}{2R^2}\right) \qquad (23)$$

The second term on the right hand side is a constant and can be combined with C so that if

$$C_1 = C - \tfrac{1}{2}\sin^{-1}(-1)$$

we have

$$\alpha + C_1 = \left(\frac{180}{\beta} + \tfrac{1}{2}\right) \cos^{-1} \frac{2R^2 - r^2}{2R^2} \qquad (24)$$

This gives

$$1 - \frac{r^2}{2R^2} = \cos \frac{\alpha + C_1}{\frac{180}{\beta} + \frac{1}{2}}$$

or

$$r = 2R \sin \frac{\alpha + C_1}{2\left(\frac{180}{\beta} + \frac{1}{2}\right)} \tag{25}$$

C_1 is determined from the condition that $r = r_1$ when $\alpha = o$ and is found to be

$$C_1 = \left(\frac{180}{\beta} + \frac{1}{2}\right)\theta_1 \tag{26}$$

The angle α_2 from the least to the greatest radius is found to be

$$\alpha_2 = 180 + \frac{\beta}{2}$$

from equation (19). The expression (21) for the profile of the remainder of the cam simplifies to

$$r' = 2R \sin \frac{C_1' - \alpha'}{2\left(\frac{180}{\beta} - \frac{1}{2}\right)} \tag{27}$$

with

$$C_1' = \left(\frac{180}{\beta} - \frac{1}{2}\right)\theta_2 \tag{28}$$

BUREAU OF METALLURGICAL RESEARCH,
CARNEGIE INSTITUTE OF TECHNOLOGY,
PITTSBURGH, PENNSYLVANIA,
DECEMBER 22, 1924.

The K-absorption-frequencies of the elements from Mg (12) to Fe (26).—A tabulation complete except for Sc (21) derived from the work of Lindh and of Fricke. In addition there is an extensive study of these frequencies as displayed by the elements Si, Ti, V, Cr, Mn, Fe in a great number of different states of chemical combination, all resulting in the conclusion that the greater the number of the outer electrons of an atom involved in binding it to another atom or atoms, the greater the amount of energy required to extract one of its innermost electrons—as though the shielding action of the outer electrons, screening the nucleus from an electron already pulled part-way out, were less effective when they are implicated in a chemical combination. There is also evidence for some altogether free atoms among the molecules of the compound.—[A. E. Lindh, Lund: Phys. ZS. *31*, pp. 210-218; 1925].

KARL K. DARROW

THE SITUATION OF AMERICAN SCIENTIFIC APPARATUS*

By H. N. Ott

Several years ago when I visited the University of Virginia, I was impressed with the serpentine brick wall surrounding the campus. Upon inquiry I found that this wall was built by Thomas Jefferson; that it was but one brick thick; that it was built in this way to save bricks because they were so expensive, it being necessary to import them from England. Bricks! imported from England after Englishmen had been on this particular territory for nearly two hundred years!! It hardly seemed possible in the light of all the resources in clay which later years have developed here at home. Yet this is the story which was solemnly told me by men whom I thought ought to know. If true, we may rightly conclude that, so far as bricks are concerned, our forefathers must have been asleep. It is a sure guess that they are now. Let us let them rest.

We are interested in the present and in the question of more vital importance as to whether or not our children a hundred years hence are going to look back on us and wonder at our short sightedness so far as scientific instruments—and possibly attainments—are concerned.

When I was a young man in college nearly everything in the laboratory—even to flasks and beakers, was imported;—and, worse than that, there seemed to be a kind of atmosphere about the place, if not directly stated, that apparatus was no good unless it were imported. We used foreign apparatus, read foreign literature, accepted foreign theories and statements almost as though decreed by the gods. I am not quarrelling so much with the fact that we paid our respects to the foreign apparatus and ideas, but that we failed to pay proper deference to our own. I am glad that our ideas as to the relative values of the two have been changing, and changing rapidly during the past ten years. With all due respect to our foreign friends and their efforts, we in this country must arrive at a point where we are at least independent and self-sufficient as regards literature, apparatus and technical ability, and I believe we shall miss our rightful heritage if we do not now seize the present opportunity to become unquestionably the leaders of the World in these things. It will mean a closer co-operation between the

* Remarks made at a Meeting of "The Apparatus Makers and Users," National Research Council.

users of scientific apparatus and the makers thereof, and above all it means a more general infusion of the ideas, among our people as a whole, as to the importance of these things. It is a most hopeful sign that business and professional men,—and many women of advanced ideas, are taking a real interest in just this thing. Business mens' conventions are providing serious talks on scientific subjects, substituting them for mere entertainment. Scientific Societies are being formed and are interesting themselves in these developments. This is, indeed, encouraging, but more can be done.

Only a few years ago the scientist was looked upon as a sort of unusual individual who, with a few tubes, bottles and reagents got results which were beautiful or wonderful to look at or tell about, with scarcely an idea that the results of his work might be vital to the comfort and well-being of the people in general. If he were an entomologist, he was just a "bug hunter" who had reached his goal when he impaled a certain number of insects on so many square inches of cork. We are beginning to see the practical application of these things, but we have not taken in the whole vision yet, nor can we. I fear that we, as an American people, have not seen how much there is in it for us, and how vital it is to our social and economic progress.

It took a war to show us how insufficient and dependent upon other people we were. Not that we were deficient in man power or in natural resources, not so much that we were deficient in mechanical wealth, not so much that we were deficient in scientists and scientific information, but that we were woefully deficient in technical and scientific apparatus. When cut off from the European supply our scientists were at their wits ends to find something to substitute. In the development of our optical glass we were compelled to buy one second hand piece of European apparatus and borrow another that was loaned us under pressure of the war emergency. We are manufacturing both now,—and other new apparatus as well. Other American manufacturers are doing the same, but we are not branching out into new equipment as we should, largely because we are handicapped by labor conditions over which we have no control;—high wages here, very low wages abroad. Nevertheless, if we accomplish what we should the manufacturers and the man interested in scientific research must stand together. Each must give and take for the benefit of both.

The last war was fought on a scientific basis. The next will be more so. At the worst, wars are only periodic. There is an economic contest going on all of the time, and here too, the winner wins only insofar as

he applies the results of scientific research to his production. Are we to allow the European scientist to come out ahead of our own with the new discoveries, or the European producer to come onto the world market ahead of our own with a new article because the European scientist and apparatus maker have together worked out some technical device by which our competitors beat us to the market?

We manufacturers must co-operate more with the research man. He, in turn, must be patient in explaining to us his ideas; and furthermore, under the circumstances, we must ask him to favor us in those things which we regularly manufacture, all other things being equal. It is most decidedly a case of working together for a common cause.

Pursuant to a very wise policy on the part of our Army by which they are continuously keeping track of sources of supply in case of an emergency, a young officer called on us to see what we might be able to do with certain pieces of apparatus, some of which we regularly make, some of which would be special, practically all of which are optical. After we had gone over each article carefully, and I had told him what we could do, he seemed very well satisfied until I told him that there was just one condition under which we could make these instruments, and that was the condition that we had the optical glass. He was surprised for he had assumed that we had on hand, and could make, the optical glass. Up to date his assumption is correct, but under present conditions nobody knows how long it will be correct. It all depends on how long the two remaining manufacturers of optical glass are going to be satisfied to face a deficit in that particular department, or whether or not some sort of a subsidy is provided by the Government, or some of the large Foundations. European countries have found it necessary. Shall we escape or shall we again depend on Europe and find ourselves again in the position in which we found ourselves only eight years ago? During the emergency stress four firms made optical glass;—two have quit, and two more would have been thousands of dollars ahead if they had quit also.

This matter of the optical glass would not be so serious if it were not so vital. I have tried to show how important the development of scientific apparatus is to our general welfare as a nation. Here, again, I want to point out the importance of optical glass in the scientific apparatus field. To be sure not all scientific apparatus is optical, but optical parts and instruments are playing a more and more important part in scientific equipment. Without optics the physicist and astronomer could not make their delicate measurements, and many of their

modern discoveries. The chemist is substituting the refractometer, the colorimeter, the spectrometer, etc., for the longer and more tedious processes of test tube analyses. We all know what the microscope and its accessories have done for the biological and medical sciences. Half of you who read this article could not do it if it were not for the optical instruments by which your eyes were properly fitted to say nothing of the spectacles themselves.

Furthermore, none of the instruments mentioned above are perfect. They will all be improved. It is my opinion that so far as the optics are concerned the greatest improvement will come along the line of the development of new glasses for new and special purposes. Shall we do it? Or, shall we let Europe do it and reap the benefit of discovery while we bring up in the rear?

Optical glass is the keynote of our National Defense. What is a battleship,—or our artillery,—worth without their range finders and other fire control instruments nearly all of which are optical? *Nothing.* The air service is also dependent.

In face of all this it may seem strange to you that even our Army and Navy are importing the optical glass they are now using. At least, they are not buying it from American firms. Of course, their appropriations are small as compared with their needs as they see them, and they feel that they cannot afford to make the sacrifice which ought to be met in some other way. The commercial purchaser buys as cheaply as he can. Some glasses can be imported for half the price of manufacture here. That is, they can be imported *now*. How about an emergency? How about any American research and development which may decide whether we take a position in the forefront, or in the rear ranks? I put the questions to you. We all want to attain results by which we, as a people, shall benefit and of which we shall be proud.

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